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Fuel Cells

HERMAN A. LIEBHAFSKY AND LEONARD W. NIEDRACH

The Thomas Precession

HENRY ZATZKIS

Dynamic Response of Large Complex Systems

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The Accuracy of Impedance Measurements in A.C. Bridges

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VOL. 269 NO. 4

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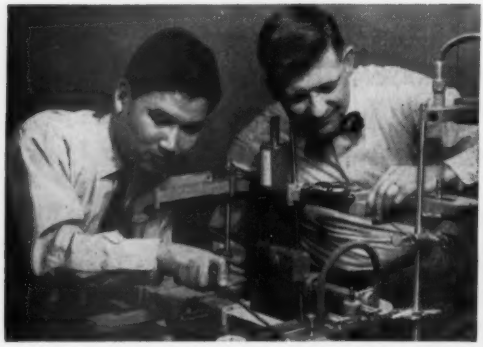
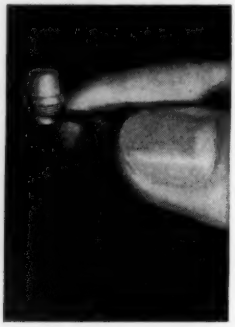


At Bell Laboratories, M. Uenohara (left) adjusts his reactance amplifier, assisted by A. E. Bakanowski, who helped develop first suitable diode.

First practical diode for amplifier as developed by A. E. Bakanowski and A. Uhlir.



How basic scientific ideas develop in the light of expanding knowledge is strikingly illustrated by the development of Bell Laboratories' new "parametric" or "reactance" amplifier.

Over 100 years ago, scientists experimenting with vibrating strings observed that vibrations could be amplified by giving them a push at strategic moments, using properly synchronized tuning forks. This is done in much the same way a child on a swing "pumps" in new energy by shifting his center of gravity in step with his motion.

At the turn of the century, scientists theorized that *electrical* vibrations, too, could be amplified by synchronously varying the *reactance* of an inductor or capacitor. Later amplifiers were made to work on this principle but none at microwave frequencies.

Then came the middle 50's. Bell Telephone Laboratories scientists, by applying their new transistor technology, developed semiconductor diodes of greatly improved capabilities. They determined theoretically *how* the electrical capacitance of these new diodes could be utilized to amplify at *microwave* frequencies. They created a new type of microwave amplifier that has far less "noise."

The new reactance amplifier has a busy future in the battle with "noise." At present, it is being developed for applications in tropospheric transmission and radar. But there are many other possible applications, including the reception of signals reflected from satellites. It is still another example of how Bell Laboratories works for better communications.

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Vol. 269 APRIL, 1960 No. 4

FUEL CELLS *

BY

HERMAN A. LIEBHAFSKY [1] AND LEONARD W. NIEDRACH [1]

ABSTRACT

A general discussion of fuel cells points out the great current interest in them and outlines the requirements they must meet. The status of representative fuel cells for special applications is indicated, and the ion-exchange membrane cell is discussed.

INTRODUCTION

A fuel cell is an *electrochemical device* in which the chemical energy of a *conventional fuel* is converted *directly and usefully* into *low-voltage direct-current* electrical energy.

We owe Benjamin Franklin a justification of this definition. Why "electrochemical device" instead of his simple "battery"? Franklin seems to have chosen "battery" because the sudden discharge of a bank of capacitors reminded him of an artillery salvo (1).[2] Today the word used in this sense refers to electrochemical cells. Such batteries, both primary and secondary, can deliver large amounts of electricity in sudden discharges because they convert chemical energy stored within them. The fuel cell by contrast is the converter alone. Fuel and oxygen must be brought to it as they must be brought to a central station generating electricity. Consequently, the fuel cell cannot be expected to provide electricity in as great surges as can batteries, and it will appear to greatest advantage over batteries when loads are moderate and long continued.

Franklin would have thought "conventional fuel" a sensible restriction. For present purposes, conventional fuels are the fossil fuels,

* This paper contains the substance of an address by the senior author at a joint meeting of The Franklin Institute and the Physics Club of Philadelphia at the Institute on December 2, 1959.

[1] General Electric Research Laboratory, Schenectady, New York.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors to the JOURNAL.)

naturally occurring gaseous hydrocarbons included, or substances easily derived from them. Hydrogen qualifies if it is not so pure as to be prohibitive in cost; so does carbon monoxide, which can be made from coal as water gas containing hydrogen, or as producer gas by the reaction of carbon dioxide. Fuels obtained by the easy rearrangement or oxidation of naturally occurring hydrocarbons, such as other hydrocarbons or methyl alcohol, must also be included. The oxidant will be oxygen, either as the element or as air.

We feel that Franklin would have supported "directly and usefully." "Directly," because it implies that the device has an anode at which fuel is oxidized and a cathode at which oxygen is reduced, this being the most direct way to convert chemical into electrical energy. "Usefully," because it indicates that we are not concerned with a toy—that we are speaking of voltages approaching the maximum permitted to us, and of current densities high enough to be practical.

Franklin might have thought "low-voltage direct-current" unnecessary in our definition because even he could scarcely have foreseen the need of emphasizing the difference between this and the high-voltage alternating-current electricity commonly distributed today. The electrochemical industry is exceptional in being a large customer for low-voltage direct-current electricity.

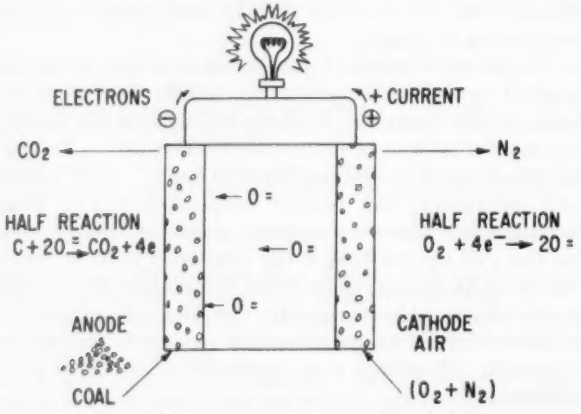

FIG. 1. Schematic carbon-oxygen direct fuel cell.

The simplest fuel cell, Fig. 1, is a visualization of an old dream, "electricity direct from coal." Here coal could be supplied at the anode, where it would interact with oxide ions to form CO_2 and release electrons to the external circuit. These electrons would do work on the way to the cathode, where they would be captured by oxygen from the air. The oxide ions thus formed would complete the circuit by flowing through the electrolyte to the anode. With oxide ions carrying

all the current, and with their production and consumption in balance, the electrolyte would remain invariant.

The compelling power of the dream visualized in Fig. 1 originates in the directness of the fuel cell. There is nothing random about the principal processes in the ideal fuel cell. The electrons want to leave the fuel at the anode and to be captured by oxygen at the cathode, and we make them do work en route. Contrast this simplicity with what happens in our present central stations. The chemical energy is completely randomized as heat when the fuel is burned under the boiler. But work is a directed process, and to acquire needed direction (which in the fuel cell is never lost), steam must be put through a guided expansion, as in a turbine. Carnot discovered the price that must be paid when heat (random) is converted into work (directed). The fuel cell escapes this penalty. A quantitative discussion of the fuel cell and the Carnot-cycle limitation (**2, 3**) is given elsewhere. Though the matter is still of vital importance, it must yield space here to other considerations that have arisen as the modern fuel cell problem continues to grow more complex.

We must leave Fig. 1 because there are other fuel cells more promising. Carbon can be made to react rapidly only at very high temperatures, and complete oxidation to CO_2 is not simple or easy. Coal contains much that is not carbon. Oxide ions do not carry current readily at low temperatures. Though carbon remains a fascinating fuel for laboratory study because the potential attractiveness of Fig. 1 is undiminished, other fuel cells have a greater likelihood of early realization. There is a diversity of fuel cells and a diversity of applications, and the situation changes almost daily. This makes exposition difficult and uncertain, but it virtually assures the success of fuel cells in restricted applications, if not across the board.

HISTORY

Within fifty years of Franklin's death, Grove (**4, 5**) hoped "to effect decomposition of water by means of its composition," and he realized his hope in investigations that showed a keenness of insight and a simple directness reminiscent of Franklin. Today we should say that Grove used hydrogen fuel cells in series to electrolyze water. These inverse electrochemical processes are being considered in energy storage schemes today.

Grove's fuel cells were platinum strips in sulfuric acid as electrolyte with hydrogen at the anode and oxygen at the cathode. He had this to say: "As the chemical or catalytic action . . . could only be supposed to take place, with ordinary platina foil, at the line or water-mark where the liquid, gas, and platina met, the chief difficulty was to obtain anything like a notable surface of action." We are struggling with this difficulty today.

For a hundred years after its origin, the fuel cell was the object of intermittent attention. Mond and Langer (6) demonstrated a more elaborate hydrogen fuel cell that developed 2 watts per square foot, in which perforated, platinized platinum-foil electrodes were pressed against absorbent layers saturated with sulfuric acid. An important address by Ostwald (7) shows that freedom of the fuel cell from the Carnot-cycle limitation was clearly understood and thoroughly appreciated in 1890. A lifetime of devotion to the fuel cell by Baur ushered in the present period of unprecedented research and development activity in the field, and the review by Baur and Tobler (8) will always be one of the classics in fuel-cell literature. Among modern investigators, Bacon (9) is distinguished for having worked on the fuel-cell problem almost continuously since 1932, and for having appreciated the importance of its engineering aspects. His hydrogen fuel cell drove a fork-lift truck in August 1959.

Examples cited later will add to this necessarily brief and incomplete history.

ADVANTAGES AND CLASSIFICATION OF FUEL CELLS

Fuel cells are attracting much greater effort and attention today than ever before in their long history. For this we may thank the growing complexity of our civilization and such of its concomitants as space-age requirements, complex military needs, congestion, and smog. The need to conserve our fossil fuels and the desirability of generating electricity at low cost are with us, as they have always been. But, for the time being at least, these matters seem less urgent than the others, and fuel cells are likely to find their first applications in some vehicle or device in which fuel economy is a secondary consideration.

We may say therefore that fuel cells promise the advantages of potentially *convenient* and *low-cost* sources of electricity, with convenience presently favored. We define convenience broadly and somewhat arbitrarily to include high power in unit volume or unit weight, quietness, cleanliness, operation on air, possibility of fuel replenishment, which makes it unnecessary to store the chemical energy in the cell. Fuel cells in which convenience is overriding in importance, and cost is secondary, we shall call fuel cells for special applications. "Special applications" as used here includes everything from locomotives to space ships—everything except central stations. Fuel cells for central stations are segregated as the limiting case in which cost is of overriding importance.

The central-station problem seems far the more difficult, another reason why it is receiving less attention at present. All but the lowest-cost fuels are necessarily excluded from central stations. But the lowest-cost fuels are just those most difficult to use practically in a cell. Coal has already been discussed in connection with Fig. 1. Heavy

heating oils aren't much better. Natural gas (methane) is notoriously inert near room temperature. So is carbon monoxide, a principal constituent of water or producer gas, both of which are mixtures readily made from coal. A successful central-station fuel cell seems so far away that we may safely dismiss it here.

PERFORMANCE REQUIREMENTS

We have already pointed out that fuel cells for special applications must meet special requirements—for example, maximum power for unit weight. Let us consider here only the performance requirements—not the special requirements—of an ideal fuel cell. The performance requirements may be simply stated: maximum performance under practical conditions for maximum time. Let us translate them into scientific terms.

Consider the fuel-cell reaction

$$\text{Fuel} + \text{oxygen} = \text{oxidation products.} \qquad (1)$$

Reaction 1 is intended to apply to any fuel we shall discuss, and always to 1 mole of the fuel (for example, 12 grams of carbon or 2 grams of hydrogen).

Let this reaction proceed to deliver current I for time Δt at the actual electromotive force E_a. (We shall ignore the fact that E, I, and Δt—the time during which 1 mole of fuel is consumed—are dependent variables.) Then

$$\text{Useful work} = -E_a I \Delta t \text{ (sign arbitrary).} \qquad (2)$$

"Practical conditions" mean that the current density I/A (A is electrode area) must be reasonably high, for otherwise we should have to build the fuel cell impracticably large. We may now say that we want high useful work for a long time from unit electrode area.

It is immediately obvious that the cell should have low internal resistance so that I^2R losses are minimized. This condition is favored by high conductivity in the electrolyte and by close spacing of the electrodes. The point requires no discussion.

The remaining important performance requirements are conveniently grouped under two headings: *reactivity requirements* and *invariance requirements*.

Reactivity Requirements

An ideal fuel cell will satisfy the following reactivity requirements:

1. Proper Stoichiometry

H_2O and CO_2 only reaction products

2. High and Unimpeded Electrode Reactivity

$$\text{High current density}$$
$$E_a/E_r \simeq 1.$$

It will be noticed that the reactivity requirements relate to the rates and mechanisms of electrode reactions. In more detail:

1. Practical fuel cells have maximum voltages not much different from 1 volt. Let us disregard voltages for the moment in discussing proper stoichiometry. The quantity of electricity from one mole of fuel, $I\Delta t$, will not be a maximum unless the fuel is oxidized to CO_2 and/or H_2O. For example, carbon oxidized to CO yields only 2 Faradays per mole instead of the 4 that pass through the circuit if CO_2 is produced at the anode. Therefore, other things equal, only half as much useful work will be done in the former case. Maximum performance requires proper stoichiometry.

2. High and unimpeded electrode reactivity will naturally bring high current density with it. But it will also make possible (if I^2R losses are low), a close approach of E_a to E_r, the maximum electromotive force possible, which is attained only in a cell operating reversibly.

Invariance Requirements

We have already said that the fuel cell is solely a converter of energy. If the converter to start with is as good as we can build, we want it to remain invariant, which should ensure long life. As invariance requirements we list:

1. No corrosion or side reactions
2. Invariant electrolyte
3. No change in electrodes

The two sets of requirements are related. If, for example, one must go to high temperatures to obtain high reactivity, it will usually be more difficult to keep the fuel cell invariant.

The reactivity requirements are the more difficult to meet. They involve the rates and mechanisms of electrode reactions. When Grove spoke of the difficulty of obtaining "anything like a notable surface of action" he was in fact describing the second reactivity requirement. Modern attempts to increase electrode reactivity rely upon increasing pressure and temperature, upon increasing surface, and upon the use of catalysts. The subject must be dismissed with two general remarks: (1) The means of increasing electrode reactivity are the means used to promote contact catalysis. But the analogy between the two cases

must not be carried too far. In the fuel cell, the catalyst is always part of an electrical circuit, and this means that opportunities exist for electron transfer that are absent in a purely thermal reaction. (2) The mixing of gases to promote reactivity at anodes deserves more attention. It is well known that the admixture of water vapor should increase the rate at which hydrocarbons oxidize at an anode, but the possibility that other gases might be helpful does not appear to have been studied.

Instead of speaking of increasing electrode reactivity, we might have spoken of reducing polarization. In either form, it is clear that we are referring to a problem that is a principal concern of electrochemists. There has been some tendency to regard the fuel cell as something mysterious. It is not. Its problems are the problems of electrochemistry, but this science has unfortunately not progressed to the point where an empirical approach to the fuel cell problem can safely be abandoned.

EFFICIENCY

In comparing the fuel cell with other energy converters (thermionic converter, thermoelectric generator) in which high efficiency is difficult to attain, there has naturally been some tendency to regard the fuel cell as superior because of the high efficiencies it promises. To counteract unwarranted optimism and to emphasize that the present treatment of the fuel cell is only introductory, we shall briefly discuss fuel-cell efficiency.

Of the various kinds of fuel-cell efficiency, we shall select the one most useful for comparison with that of a central station. It is

$$\eta_a = \frac{E_a I \Delta t}{\Delta H}. \tag{3}$$

The numerator is the useful work (Eq. 2) and the denominator is the heat of reaction for Reaction 1; we may take this to be the higher heating value of 1 mole of fuel. The best modern central stations, being Carnot-cycle limited, operate at efficiencies near 40 per cent.

The fuel cell, which escapes this limitation, can rather easily show greater efficiencies. It therefore is bound to appear to good advantage when it is compared on this basis with any Carnot-cycle-limited converter. But it is well to remember that 90 per cent efficiency is useless if the current density at this efficiency is prohibitively low or if the life of the fuel cell is very short. (In general, efficiency decreases with increasing current density.) In other words, efficiency means little unless current density also is specified and unless other necessary requirements are also met.

One must not, however, depreciate high efficiency out of hand. If, for example, the fuel cell serves a space ship in which pay load is at a

premium, one may prefer to tolerate low current density in order to decrease the weight of fuel carried. The amount of heat liberated during the operation of a fuel cell is lower at high efficiencies. One may prefer to tolerate lower current densities in order to simplify the problem of disposing of rejected heat.

The kind of considerations just described govern the choice of every important fuel-cell parameter. A successful design is always a compromise made especially for the application in question.

FUEL CELLS FOR SPECIAL APPLICATIONS

Approximate characteristics are given in Table I for a number of fuel cells intended for special applications.

TABLE I.—*Approximate Characteristics of Representative Fuel Cells for Special Applications.*[a]

	Electrolyte	Fuel	Electrodes	Operating Temp., °C.	Operating Pressure	Current Density at 0.7 V.[b] amps/ft²
Allis-Chalmers Co. (10)	—	A gas mixture—largely propane	—	Ambient	Atmospheric	~20
Bacon (9)	Aqueous KOH	Hydrogen	Porous nickel	200° C.	400–600 PSI	400
Esso Research & Engineering Co. (11)	—	Ethane	—	—	—	—
General Electric Co.	Strong acid Ion Exchange Membrane	Hydrogen	Pt or Pd	Ambient to 80°	Atmospheric	15–35
Justi (12)	Aqueous KOH	Hydrogen	Porous metal (Ni; Ag)	85°	Atmospheric	>400
National Carbon Co. (13)	Aqueous KOH	Hydrogen	Porous carbon with catalyst	50–70°[c]	Atmospheric	>250
Russian (14)	Aqueous KOH	Hydrogen	Porous carbon with catalyst	Ambient	Atmospheric	10–20

[a] Cells operating above 300° C. are excluded.
[b] Open circuit voltages are 1.0–1.1 volt in all cases.
[c] Self-starting from 20° C.

THE ION-EXCHANGE MEMBRANE CELL

Some years ago, Grubb (**15, 16**) introduced ion-exchange membranes as electrolytes in various devices, fuel cells included. In acid membranes, hydrogen ion carries all the current. Niedrach (**17**) succeeded in raising the current density available in hydrogen fuel cells with such electrolytes. Although these cells are still being investigated, they can serve as illustrative examples for the reactivity and invariance requirements. Figure 2 shows the cell in section, and Fig. 3 contains a representative polarization curve.

Stoichiometry first. The reaction products appear at the cathode. Although the overall reaction results in the formation of water, buildup

of a trace amount of hydrogen peroxide as an intermediate may be associated with the activation polarization observed.

The polarization curve shows that the reactivity requirements are met only moderately well. Activation polarization is present, and special experiments show it to be mainly at the cathode. In an attempt to minimize this kind of polarization, the noble metals palladium and

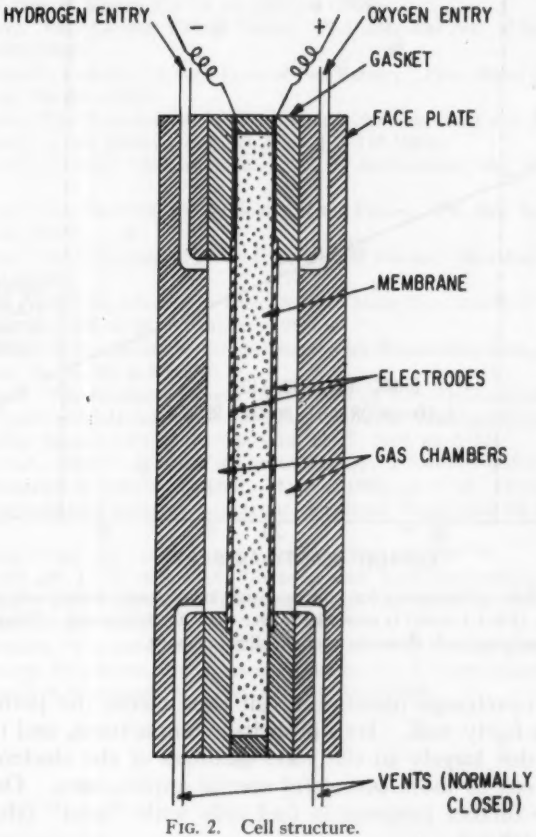

FIG. 2. Cell structure.

platinum have been chosen as electrode materials. Special measures were necessary to obtain intimate contact between electrode metal and ion-exchange membrane; this intimate contact is necessary if there is to be the "notable surface of action" sought by Grove as prerequisite to the desired high current density.

With liquid electrolytes, the "notable surface of action" is realized by using highly porous electrodes. These are liable to fill with the

electrolyte or with water formed as reaction product. The electrodes then "drown"—that is, they are not invariant. The ion-exchange membranes escape this danger because they reject the water as it forms on the oxygen electrode. The fact that hydrogen ion carries all the current in acid membranes also makes for invariance.

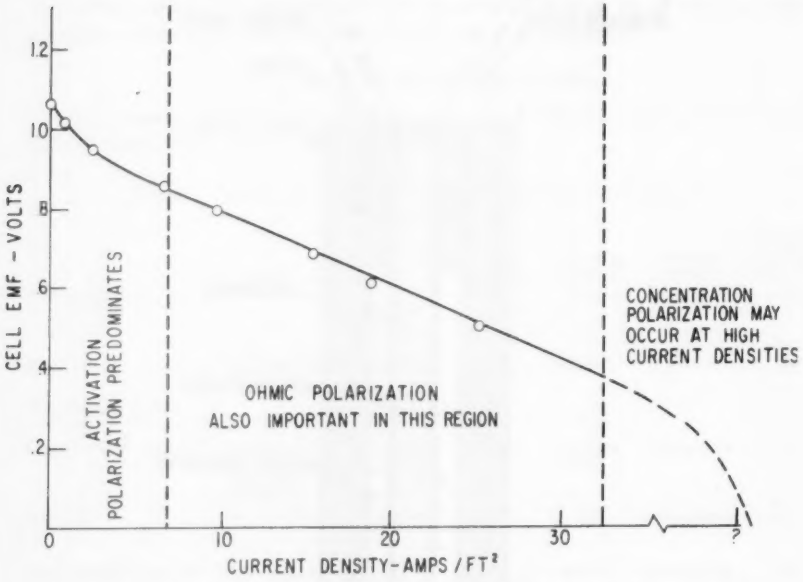

FIG. 3. Polarization curve for representative ion-exchange membrane cell. The open-circuit voltage (1.0–1.1 volts) is somewhat lower than E_r (1.23 volts). Kind of polarization important in each region is shown in over-simplified manner.

The ion-exchange membrane cell thus meets the performance requirements fairly well. Its simplicity, compactness, and convenience, which are due largely to the good qualities of the electrolyte, should find for it one or more successful special applications. One may look forward to further progress in fuel cells with "solid" (that is, quasi-solid) electrolytes.

CONCLUSION

The current increase in fuel-cell activity, if maintained, makes it likely that fuel cells will serve as power sources in special applications within the next five years. Successful, practical model cells are already with us. The future of central-station fuel cells cannot be predicted today.

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THE THOMAS PRECESSION

BY

HENRY ZATZKIS [1]

ABSTRACT

The equation for the Thomas precession is derived by using simple matrix calculation. No knowledge of group theory of Lobachevski geometry is assumed.

Most textbooks on the theory of relativity pay scant attention to the Thomas precession although this purely kinematic phenomenon is just as important a consequence of the Lorentz transformation as the Einstein addition law of velocities. The reason for this neglect is probably that sophisticated group-theoretic manipulations or the apparatus of Lobachevski trigonometry[2] are deemed necessary to derive the equation for the Thomas precession.

The fundamental cause of the Thomas precession is the following: While it is true that the Lorentz transformations linking all reference systems moving along a common axis and coinciding at the time $t = 0$ form a group, this is no longer true if the directions of their relative motions are not parallel to each other. Specifically, if system S' moves with respect to S along the positive x-axis of S with speed v_1, and if system S'' moves with respect to S' along the positive y'-axis of S' with speed v_2, and if the origins coincide at $t = 0$, the transition from S to S'' is *not* a pure Lorentz transformation, but a Lorentz transformation combined with a rigid rotation. It is this latter rotation which is responsible for the Thomas precession. The object of this paper is to prove the existence of this rotation and to derive from it the Thomas precession.

The following abbreviations will be used:

$$\beta_1 = \left(1 - \frac{v_1^2}{c^2}\right)^{-1/2} \tag{1a}$$

$$\beta_2 = \left(1 - \frac{v_2^2}{c^2}\right)^{-1/2} \tag{1b}$$

$$\beta = \left(1 - \frac{w^2}{c^2}\right)^{-1/2}. \tag{1c}$$

[1] Newark College of Engineering, Newark, N. J.

[2] C. Møller, "The Theory of Relativity," New York, Oxford University Press, 1952, p. 53; W. H. Furry, *Am. J. Phys.*, Vol. 23, p. 517 (1955); and A. Sommerfeld, "Atombau und Spektrallinien," Vol. I, fifth edition, Vieweg, Braunscheig, 1931, p. 709.

The transformation law that connects S and S' is given by[3]

$$\begin{bmatrix} \beta_1 & 0 & 0 & -v_1\beta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dfrac{-v_1\beta_1}{c^2} & 0 & 0 & \beta_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix}. \tag{2}$$

The transformation from S' to S'' is given by

$$\begin{bmatrix} 1 & 0 & 0 & 0 & x' \\ 0 & \beta_2 & 0 & -v_2\beta_2 & y' \\ 0 & 0 & 1 & 0 & z' \\ 0 & \dfrac{-v_2\beta_2}{c^2} & 0 & \beta_2 & t' \end{bmatrix} = \begin{bmatrix} x'' \\ y'' \\ z'' \\ t'' \end{bmatrix}. \tag{3}$$

The transformation from S to S'' is therefore given by the product of the two matrices:

$$\begin{bmatrix} \beta_1 & 0 & 0 & -v_1\beta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ \dfrac{-v_1\beta_1}{c^2} & 0 & 0 & \beta_1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \beta_2 & 0 & -v_2\beta_2 \\ 0 & 0 & 1 & 0 \\ 0 & \dfrac{-v_2\beta_2}{c^2} & 0 & \beta_2 \end{bmatrix} = \begin{bmatrix} \beta_1 & 0 & 0 & -v_1\beta_1 \\ \dfrac{v_1v_2\beta_1\beta_2}{c^2} & \beta_2 & 0 & -v_2\beta_1\beta_2 \\ 0 & 0 & 1 & 0 \\ \dfrac{-v_1\beta_1\beta_2}{c^2} & \dfrac{-v_2\beta_2}{c^2} & 0 & \beta_1\beta_2 \end{bmatrix} \tag{4}$$

It is convenient to introduce at this point abbreviated notations. We shall denote the matrix on the right-hand side of Eq. 4 by A. We shall also denote the column vectors

$$\begin{bmatrix} x \\ y \\ z \\ t \end{bmatrix} = \vec{X} \quad \begin{bmatrix} x' \\ y' \\ z' \\ t' \end{bmatrix} = \vec{X}' \quad \begin{bmatrix} x'' \\ y'' \\ z'' \\ t'' \end{bmatrix} = \vec{X}''. \tag{5}$$

Our fundamental assertion can now be formulated: It is possible to write the matrix A as a product of two other matrices B and C, that is

$$A = BC \tag{6}$$

where C is a proper Lorentz transformation, and B is a rigid rotation about the common z-axis.

[3] E. Madelung, "Die Mathematischen Hilfsmittel des Physikers," third edition, Berlin, Springer, 1936, p. 272.

Proof:

Preliminary to the proof we make the following three observations: (*a*) It is a direct consequence of the Einstein addition law for velocities that the reference frame S'' has the velocity $\overline{w}$ with respect to S where the components of $\overline{w}$ are

$$w_x = v_1 \tag{7a}$$

$$w_y = \frac{v_2}{\beta_1}. \tag{7b}$$

From (7*a*) follows

$$w^2 = w_x^2 + w_y^2 = v_1^2 + v_2^2 - \frac{v_1^2 v_2^2}{c^2} \tag{7c}$$

$$\beta = \left(1 - \frac{w^2}{c^2}\right)^{-1/2} = \beta_1 \beta_2. \tag{7d}$$

Utilizing the relations (7), we can rewrite

$$A = \begin{bmatrix} \beta_1 & 0 & 0 & -v_1\beta_1 \\ \dfrac{v_1 v_2 \beta}{c^2} & \beta_2 & 0 & -v_2\beta_2 \\ 0 & 0 & 1 & 0 \\ \dfrac{-v_1\beta}{c^2} & \dfrac{-v_2\beta_2}{c^2} & 0 & \beta \end{bmatrix}. \tag{8}$$

(*b*) The reference frame S'' is rotated with respect to S and has a velocity $\overline{w}$ with respect to S. We shall introduce here an intermediate frame S^* which also has the velocity $\overline{w}$ with respect to S but whose axes are still parallel to those of S. The transformation between S and S^* is given by

$$C\overline{X} = \overline{X}^* \tag{9}$$

where

$$\overline{X}^* = \begin{bmatrix} x^* \\ y^* \\ z^* \\ t^* \end{bmatrix} \tag{10}$$

and C is given by[4]

[4] See footnote 3.

$$C = \begin{bmatrix} 1 - \frac{w_x^2}{w^2}(1-\beta) & -\frac{w_x w_y}{w^2}(1-\beta) & 0 & -w_x\beta \\ -\frac{w_x w_y}{w^2}(1-\beta) & 1 - \frac{w_y^2}{w^2}(1-\beta) & 0 & -w_y\beta \\ 0 & 0 & 1 & 0 \\ -\frac{w_x\beta}{c^2} & -\frac{w_y\beta}{c^2} & 0 & \beta \end{bmatrix}. \tag{11}$$

We shall need the inverse of C. It can be written down at once from the consideration of its physical meaning. It is merely necessary to change the signs of w_x and w_y (note that the determinant of C has the value 1).

$$C^{-1} = \begin{bmatrix} 1 - \frac{w_x^2}{w^2}(1-\beta) & -\frac{w_x w_y}{w^2}(1-\beta) & 0 & w_x\beta \\ -\frac{w_x w_y}{w^2}(1-\beta) & 1 - \frac{w_y^2}{w^2}(1-\beta) & 0 & w_y\beta \\ 0 & 0 & 1 & 0 \\ \frac{w_x\beta}{c^2} & \frac{w_y\beta}{c^2} & 0 & \beta \end{bmatrix}. \tag{12}$$

(*c*) Since B is assumed to be a rigid rotation about the z-axis, it must be of the form

$$B = \begin{bmatrix} \cos\phi & -\sin\phi & 0 & 0 \\ \sin\phi & \cos\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{13}$$

where ϕ is the angle of rotation from S towards S^*. It is this angle that we wish to determine. The solution for B is given by

$$B = AC^{-1} \tag{14}$$

or

$$B = \begin{bmatrix} \frac{\beta_1 v_1^2 + \beta_2 v_2^2}{\beta w^2} & \frac{-v_1 v_2(\beta - 1)}{\beta w^2} & 0 & 0 \\ \frac{v_1 v_2(\beta - 1)}{\beta w^2} & \frac{\beta_1 v_1^2 + \beta_2 v_2^2}{\beta w^2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{15}$$

Since the matrix elements of B are real and satisfy the relations

$$B_{11} = B_{22} \tag{16a}$$

$$B_{12} = -B_{21} \tag{16b}$$

$$B_{11}^2 + B_{12}^2 = 1 \tag{16c}$$

we can equate

$$\cos\phi = \frac{\beta_1 v_1^2 + \beta_2 v_2^2}{\beta w^2} \tag{17a}$$

$$\sin\phi = \frac{v_1 v_2(\beta - 1)}{\beta w^2}. \tag{17b}$$

The relative orientation between S and S'' is shown in Fig. 1. The

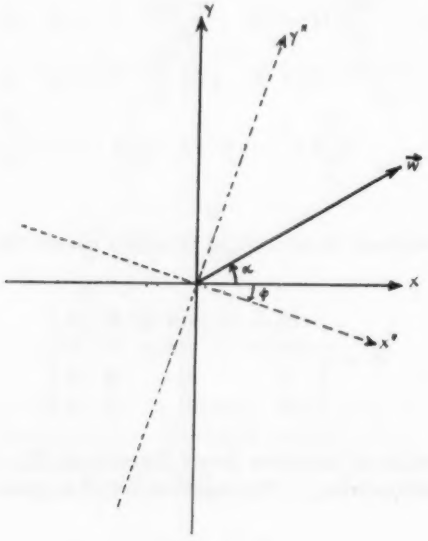

FIG. 1.

positive z-axis is pointing toward the viewer. $\vec{w}$ shows the direction of motion of S'' with respect to S.

$$\tan\alpha = \frac{w_y}{w_x} = \frac{v_2}{\beta_1 v_1}. \tag{17c}$$

Equation 17b can be rewritten

$$\sin\phi = \frac{w_x w_y(\beta - 1)}{\beta_z w^2}. \tag{18}$$

If we assume the w_y is infinitesimally small compared to w_x, then $\sin \phi$ can be replaced by $d\phi$, β_2 by 1, and w_y can be interpreted as the infinitesimal change of $\vec{w}$ in the y direction, and w_y divided by the time differential dt can be interpreted as the component of the acceleration $\vec{a}$ in the y direction.

Hence, we rewrite Eq. 18

$$d\phi = \frac{|\vec{w}|\,|\vec{a}|\,dt \sin \gamma(\beta - 1)}{|\vec{w}|^2} \tag{19}$$

where γ is the angle between the instantaneous velocity, $\vec{w}$, and the instantaneous acceleration, $\vec{a}$, or

$$\vec{\omega} = \frac{d\phi}{dt} = \frac{|\vec{w}|\,|\vec{a}|\,\sin \gamma(\beta - 1)}{|\vec{w}|^2}. \tag{20}$$

From Fig. 1 we notice that ω is the magnitude of the angular velocity vector, $\vec{\omega}$, where

$$\vec{\omega} = \frac{\vec{a} \times \vec{w}}{|\vec{w}|^2}(\beta - 1). \tag{21}$$

This is the Thomas equation.

If $w \ll c$, this formula reduces to

$$\vec{\omega} = \frac{\vec{a} \times \vec{w}}{2c^2}. \tag{22}$$

Acknowledgments

It is a pleasure to thank Dr. C. W. Kilmister (Imperial College, London) and Dr. G. Stephenson (King's College, London), who suggested this approach to the problem in a letter by Dr. G. Stephenson to the author.

DYNAMIC RESPONSE OF LARGE COMPLEX SYSTEMS

BY

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I. INTRODUCTION

Study of large complex systems presents one of the major challenges of contemporary applied science. In the realm of analysis, the evolution of the performance of large complex systems of quite different nature such as physical, engineering, economical, biological, etc., becomes inevitable and every further development of the methods of analysis is of great importance. Similarly in the realm of synthesis, new methods are necessary for handling man-made systems of considerable complexity. In the last decade new branches of science have emerged, such as general systems theory, operational research, systems engineering, etc., all primarily devoted to problems in handling large complex systems.

In any study of systems behavior, statistical treatment of the phenomena becomes very important. One can cite three major reasons for that: (1) The data related to the inputs or the parameters of the system cannot be determined accurately and on a deterministic basis due to the fact that the total of the causes related to the system under consideration is not known and, further, due to the action of the uncertainty principles. The data then must be treated statistically. (2) The laws governing the behavior of the system could be unknown although the presence of these laws can be observed by their effect. The relationships might then be described statistically. (3) The number of variables associated with the system can be very large and the system as a whole can be treated successfully only using statistical methods although the precise behavior of every subsystem as well as their interrelations could be described deterministically.

This paper will be devoted to the problems in the last group. While in the first two groups of problems the statistical treatment has been introduced by necessity as the consequence of lack of knowledge of systems behavior, in the third group, statistical treatment has been introduced for convenience although the system could be handled, at least in theory, on a deterministic basis.

Many systems of interest (such as thermodynamic, organizational, biological, etc.) have such a large number of subsystems and variables that any deterministic study seems inconceivable. The statistical approach, then, appears inevitable. However, for such an approach one should assume that the system under consideration is in the state of

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"known disorder" so that the concept of probability can be associated with the phenomena. We shall refer to these systems as systems with known disorder. However, there are also systems with extremely large numbers of subsystems but in which the associated phenomena can be described on a deterministic basis. We shall refer to these systems as ordered systems.

The principal effort in many fields of systems science has been devoted to the study of systems of known disorder, the possibilities in the study of ordered systems being largely overlooked. In this paper we shall be concerned with the study of large, complex, but ordered systems. We shall give a framework for the development of a theory for study in large ordered system and discuss some of the results already obtained.

We shall be concerned with the abstract concept of the system as defined in the subsequent section. Two major contributions so far made to the study of large ordered systems have been devoted to a special kind of system—energy systems: Kron's matrix method in network study (**4**)[2] and Paynter's energy multi-port method in the study of general energy exchange (**5**). The common feature of these analyses was the validity of the laws of conservation of energy. However, our effort will be devoted to systems where matter as well as energy can be exchanged with environment and also where the system is of a non-physical kind. No conservation laws can be prescribed to such systems.

II. DEFINITION OF THE PROBLEM

We shall adopt the following definition of the system (**3**): A system is a set of interrelated concepts.

The definition is general enough to cover all kinds of systems in which we are interested, particularly the physical systems. The principal feature of a system is the presence of interrelations. A concept often relates to a set of attributes of a particular object. However, it could be defined quite abstractly as, for example, a particular feeling such as hate, or some objects as a street, car, animal, etc. When one refers to physical systems, the concept could be called object. In this case an object is defined as a bearer of the attributes (**1**). The notion of the system in abstract has been discussed elsewhere (**3**) and is beyond the scope of this paper. It will become clearer in the subsequent discussion.

The system can be divided into subsystems. In abstract theory this subdivision is arbitrary. However, the basic structure of the problem itself very often decides which parts of the system should be considered as subsystems. In this paper we shall be concerned with systems where the very essence of the problem requires the definition of a large number of subsystems and therefore such systems shall be called large. Each of the subsystems can be of arbitrary complexity and changes in any of them will be assumed to influence all others. Therefore, this class of systems shall be referred to as complex.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

The relationships between the system and the rest of the universe, called the environment, is usually expressed in terms of inputs and outputs. We shall consider the case where the numbers of inputs and outputs are large.

One of the major difficulties in the general analysis of systems is due to arbitrariness in the choice of the subdivision procedure and in the description of the relationship between the system, subsystems, and their environment, that is, definition of inputs and outputs. We shall distinguish two kinds of systems, energy-matter systems and signal systems. Energy-matter systems are physical systems and the associated variables can always be expressed in terms of matter and energy or its conjugate variables—general effort and general flow (2). The signal systems are either nonphysical or are obtained from the physical systems by abstraction as, for example, signal models used in control theory, in which some of the variables can be general efforts and some general flows depending upon our interest (7, 8).

The basic problems which we shall consider in the subsequent discussion are the following: (1) how the interrelations in a large system could be described in a unified way; (2) how the systems can be classified on the basis of these interrelations; (3) what are the properties of the class of systems defined in such a way; and (4) some techniques for experimentation with large systems will be outlined.

Analysis of a large complex system has usually been considered from one of the following two points of view: microscopic, when the behavior of particular substystems is analyzed in detail, and macroscopic, when only the behavior of the system as a whole is analyzed. Our present point of view is not covered entirely by either of the above-mentioned considerations and may be regarded as a third possibility.

This present point of view could be defined as follows: knowing the behavior (attributes) of every subsystem regarded as an isolated part, and knowning the relationships that tie the system together, what will be the behavior of the subsystems looking at them as integral parts of the whole system, and what will be the behavior of the system itself? We are thus at the same time interested in both macroscopic and microscopic views of the system's behavior.

Investigations of this kind are justified because large systems show ad limitum new properties that cannot be anticipated merely by inspection of any of the subsystems. On the other hand, for complete understanding of the behavior of the integral system, the behavior of the subsystems should also be known. This view is sometimes expressed by stating that a system is more than an ensemble of its parts (6).

We shall not treat here particular examples but rather will use the abstract mode of exposition. Applications of the theory, however, have already been fruitful in engineering systems (12) and in some studies of general systems (3).

III. THE CONCEPT OF A GENERALIZED FLUID

Classification of the subsystems is very often related to the number of inputs and outputs. Regarding this number, three different cases should be defined: (1) In the first case there are but one input and one output. (2) In the second case there are two or more, but always a finite number of, inputs and outputs. We can now subdivide the system into subsystems and define the interrelations appropriately. Due to these interrelations new phenomena in the behavior of the system appear (for example, instability due to interrelations). (3) In the third case there are, in the limit, denumerable infinite numbers of inputs and outputs so that a denumerable infinite number of subsystems can be defined. Of the primary interest in this case are the questions: what properties of the subsystems have been preserved in the interconnecting system and, further, what new properties have appeared as a consequence of an infinite (very large) number of subsystems?

We shall be concerned in this treatment only with linear systems except otherwise stated. For these systems the input-output relations can be expressed by means of a set of linear differential equations. Using functional transformations, these equations are usually written in the form

$$\begin{aligned} \phi_1 &= P_{11}\mu_1 + P_{12}\mu_2 + \cdots + P_{1n}\mu_n \\ &\vdots \\ \phi_n &= P_{n1}\mu_1 + P_{n2}\mu_2 + \cdots + P_{nn}\mu_n \end{aligned} \tag{1}$$

where

$\mu_1 \cdots \mu_n$—inputs to the system
$\Phi_1 \cdots \Phi_n$—outputs of the system
$P_{11} \cdots P_{nn}$—transfer functions of the subsystems and interconnecting elements.

With matrix notation these sets of equations can be written in the form

$$\Phi = PM \tag{2}$$

where

$$M = \begin{bmatrix} \mu_1 \\ \vdots \\ \mu_n \end{bmatrix} \qquad \text{column vector of the inputs}$$

$$\Phi = \begin{bmatrix} \phi_1 \\ \vdots \\ \phi_n \end{bmatrix} \qquad \text{column vector of the outputs}$$

$$P = \begin{bmatrix} P_{11} \cdots P_{1n} \\ \vdots \quad\quad \vdots \\ P_{n1} \cdots P_{nn} \end{bmatrix}$$

The corresponding block diagram is given in Fig. 1.

In general, the numbers of inputs and outputs are different. We shall, however, consider the case when numbers of inputs and outputs are equal. In special cases when the numbers of inputs and outputs are different, modification of the theory can be readily made.

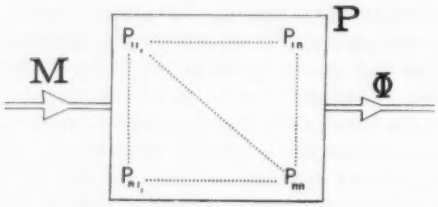

FIG. 1.

The existence of the interrelations distinguishes single input-output systems from complex systems. Therefore, an increased knowledge of how these interrelations actually behave will give us a better understanding of the complex system itself.

One of the main topics of this analysis of large complex systems is therefore a better understanding of the influence of the interrelations on the behavior of the whole system.

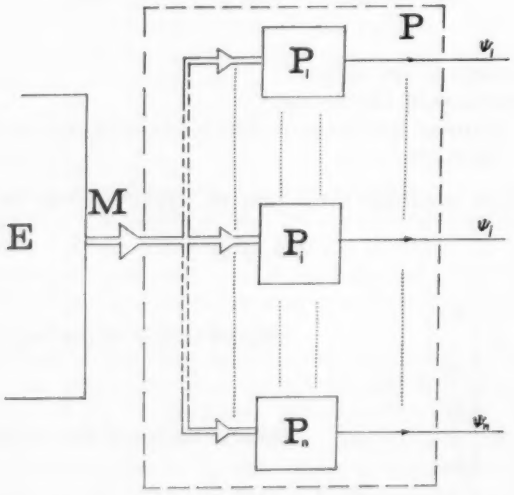

FIG. 2.

First, however, a definition of interrelated systems should be given. We shall adopt the following definition:

Substystems are to be considered as interrelated if changes in the properties in any of them influence the behavior of the outputs of other (not necessarily all) subsystems. If a change in one of the subsystems does not affect the other subsystems, it should be considered as independent (noninterrelated).

One of the characteristics of the given definition is that it refers to the subsystems and not to the set of the inputs and outputs of the system. Therefore, the subdivision procedure itself determines the kind of the subsystems. For example, in the linear case one can subdivide a system into a given number of subsystems but with different types and degrees of interactions. The most commonly used type of subdivision (as given in Fig. 1 and Eq. 2) represents the system as a set of noninteracting groups of subsystems. The complexity of the system is merely manifested by the fact that the same vector input affects all, otherwise independent, subsystems as shown in Fig. 2. The system is then characterized by means of n^2 forward acting transfer functions (Fig. 3). The change in the properties of a subsystem affects

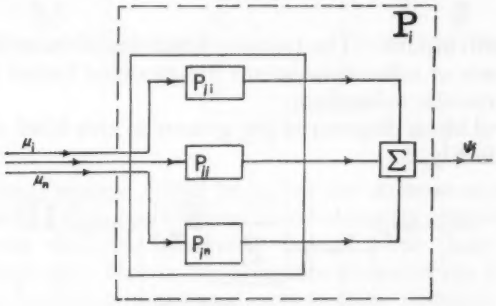

FIG. 3.

only its own output. One of the subsystems can even be made unstable by a change in its internal feedback, while all other subsystems will remain stable. Therefore, although all linear systems can be subdivided into subsystems as expressed in Eq. 2, it is apparent that this representation does not reflect actual properties of the system if the structure of the system is really of an interrelating kind.

A principle of the uncertainty in the structure is discussed in more detail elsewhere (3). Here we want only to point out that according to this principle there is an infinite number of physical systems with the same terminal behavior but whose internal structure is in principle undeterminable by means of any experiment from outside the system. An appropriate way to represent the interrelated systems is given in Fig. 4.

The interrelations are now represented by a set of elements connect-

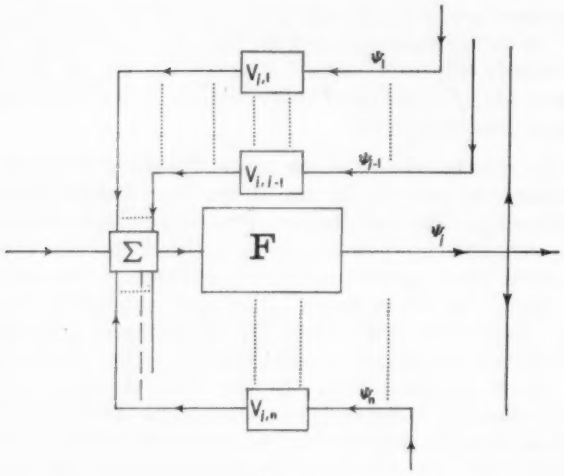

FIG. 4.

ing outputs with inputs. The transfer functions of these elements show how the outputs of other subsystems are modified before acting as the input to a particular subsystem.

The general block diagram of the system in this kind of representation is given in Fig. 5.

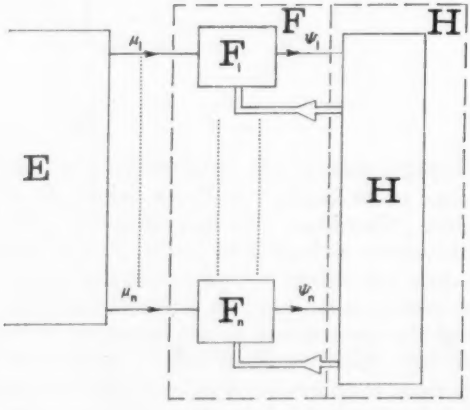

FIG. 5.

The corresponding matrix equation of the complex system is

$$\Phi = F[M - V\Phi] = F[M - \Delta M] \tag{3}$$

where

$$F = \begin{bmatrix} F_1 & 0 & \cdots & 0 \\ 0 & F_2 & & \vdots \\ \vdots & & & \vdots \\ 0 & \cdots & \cdots & F_n \end{bmatrix}$$ a diagonal matrix defining all the subsystem

$$V = \begin{bmatrix} V_{11} V_{12} & \cdots & V_{1n} \\ \vdots & & \\ V_{n1} V_{n2} & \cdots & V_{nn} \end{bmatrix}$$ a square matrix defining the interaction process.

The block diagram of the matrix representation is given in Fig. 6.

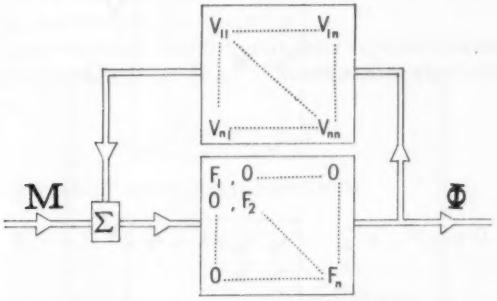

FIG. 6.

The diagonal matrix F will be called the subsystem matrix. The interaction occurs through the functional elements connecting the subsystem outputs with the subsystem inputs. For these elements we need a new concept. Due to the arbitrary choice of the input and output variables, we cannot use any conservation principle as the basis underlying analysis. But for the general analysis we need a model of the interrelated systems, which although abstract, will enable classification and thereafter a unified treatment. Therefore, for reason of generalization and conceptualization, we shall find it helpful to suppose that the outputs act on the inputs through a fluid of a general type. One of the reasons that leads us to adopt such a concept for the unified definition of interactions is that it covers almost all possible cases of continuous and discrete interaction, including both instantaneous and local action as well as retarded potential influence. Therefore, we shall refer to matrix V as the hyperfluid matrix.

Some remarks are immediately necessary to clarify the ideas. Such a generalized hyperfluid is merely an abstraction in terms of realtionships, and one does not imply the physical existence of any fluid of conventional behavior. Therefore, in transporting the influences from one subsystem to the others, the hyperfluid can modify these influences

in any way that corresponds to the actual situation. The hyperfluid need not be homogeneous either in a physical sense or in the form of relationships.

Although the above concept of the hyperfluid is general enough to cover all possible cases, what makes this concept useful is that in many practical cases, though nonhomogeneous, the hyperfluid affords relationships of a certain type for all mutual influences in a particular system. For example, it can represent molecular forces in some physical problems or magnetic forces in some electromagnetic problems; the synchronizing torques in power system enginering; fuel-air mixture in engine problems; etc.

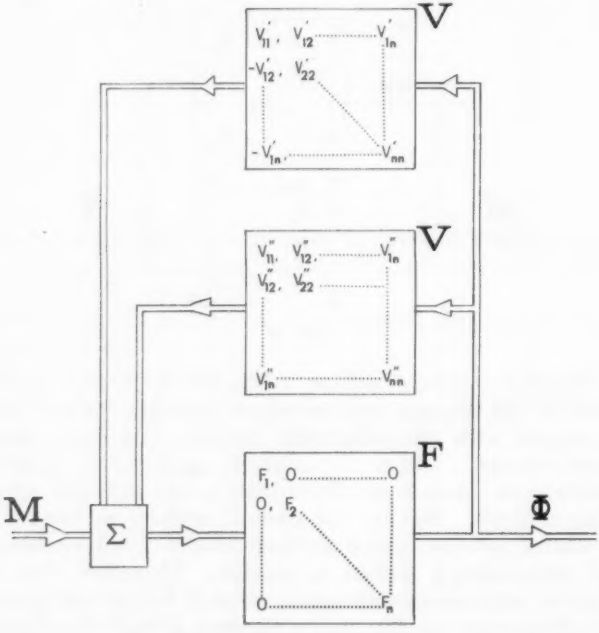

FIG. 7.

The classification of the various systems can be made now with respect to the form of the hyperfluid matrix. For example, the system can have hyperfluid with no storage (of energy, information or any other quantity under consideration) in the steady state, or even during transient; an elastic or integral type of hyperfluid; a fluid which is compressible or incompressible in the general sense, etc.

In many problems, as, for example, control of complex processes, one wants to modify the systems regarding its interrelations. Then the task could be defined by looking only at the hyperfluid matrix V.

In some problems it appears to be advantageous to use the matrix algebra and present the general fluid by means of more parallel matrices, each presenting some of the characteristic properties. For example, the hyperfluid matrix can be represented by its symmetric and skew symmetric component (Fig. 7). Furthermore, the hyperfluid matrix can be resolved in components, one of which represents only the steady state, with others representing only the transient state. Further subdivision may lead to the elastic interrelations matrix, compressible fluid matrix when every output variable in the fluid depends upon the difference of the input quantities, etc. In this way we can recognize, if there are any, the conventional properties and separate them from the special features of the problem at hand. In such a way the unified analysis can be performed.

In linear cases between the conventional representation (Fig. 2) and the hyperfluid representation (Fig. 5) there exists algebraic relationship in matrix form

$$V = P^{-1} - F^{-1} \tag{4}$$

or by means of a set of operational equations

$$F_1 = \sum_j [1 + F_1 V_{1j}] P_{1j} \qquad \sum_j V_{kj} P_{kj} = 0. \tag{5}$$

However, in nonlinear cases only one of the representations is physically justified, which corresponds to the actual behavior of the system itself.

IV. CLASSIFICATION OF THE COMPLEX SYSTEMS

Classification of large complex systems can be made by considering the general feedback matrix. We shall discuss here only some of the important special cases that present extremity with respect to the strength of the interrelations.

If the subsystems are noninteracting at all, so that physical summativity is valid, the hyperfluid matrix is diagonal

$$V = \begin{bmatrix} V_{11} 0_1 \cdots 0 \\ 0_1 V_{22} \cdots 0 \\ \vdots \qquad \vdots \\ 0 \cdots\cdots V_{nn} \end{bmatrix}. \tag{6}$$

The matrix equation for the system is now

$$\Phi = FM. \tag{7}$$

If the outputs are superimposed, the resultant output could be obtained by multiplying Eq. 7 with unity row vector D so that we obtain

$$D\Phi = DFM. \tag{8}$$

This equation is now scalar and can be written in the form

$$\sum_k \phi_k = \sum_k F_k \mu_k. \tag{9}$$

If all the inputs are of the same kind (for example, unit step), we have

$$\phi = \sum_k \phi_k = \mu \sum_k F_k = \mu F. \tag{10}$$

The resultant transfer function is the sum of the transfer functions of all the subsystems taken together.

As the second example, let us consider the systems in which the hyperfluid cannot store the input quantity. By this we mean that there exists one linear functional dependence between the inputs and outputs in which appear only the subsystem matrix elements and no hyperfluid matrix elements. For the hyperfluid elements we have now

$$\sum_k \sum_j V_{kj}\phi_j = 0. \tag{11}$$

This constraint is satisfied if between the elements of the hyperfluid matrix there exist the relationships

$$V_{jj} = -\sum_{k \neq j} V_{jk}; \qquad V_{kj} = V_{jk} \tag{12}$$

so that the matrix is symmetric

$$V = \begin{bmatrix} V_{11} V_{12} \cdots V_{1n} \\ V_{12} V_{22} \quad\; V_{2n} \\ \vdots \qquad\quad \vdots \\ V_{1n} V_{2n} \quad\; V_{nn} \end{bmatrix}. \tag{13}$$

By multiplying Eq. 3 with unity row vector, we obtain now

$$D\Phi = DFM - DFV\Phi. \tag{14}$$

Due to Eq. 12 we have

$$DV = 0 \tag{15}$$

so that we obtain

$$DF^{-1}\Phi = DM. \tag{16}$$

Because the matrix F is diagonal, its transpose can be simply obtained

$$F^{-1} = \begin{bmatrix} 1/F_1, & 0, & 0 \\ 0, & 1/F_2 & \\ \vdots & & \\ 0 \cdots\cdots & & 1/F_n \end{bmatrix}. \tag{17}$$

In scalar form Eq. 16 is

$$\sum_k \mu_k = \sum_k \frac{\phi_k}{F_k}. \tag{18}$$

Equation 18 gives the behavior of the system at its boundaries or control surface. In Eq. 18 the interrelations do not appear. In technical system it means that the interrelations are achieved by means of a compressible lossless fluid. In these cases the mean mode defined by

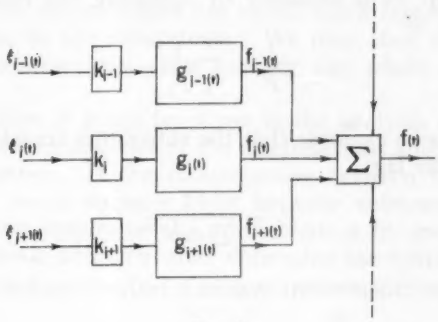

FIG. 8.

$$\phi = \frac{\sum \frac{\phi_k}{F_k}}{\sum \frac{1}{F_k}} \tag{19}$$

plays a rather important role. For the mean mode we have

$$\sum_k \mu_k = \phi \sum_k \frac{1}{F_k}. \tag{20}$$

If the outputs should be preserved, we have, in addition to Eq. 12, to assume

$$V_{jk} \to \infty. \tag{21}$$

From Eq. 3 we obtain, then,

$$\phi_1 = \phi_2 = \cdots = \phi. \tag{22}$$

This case has great theoretical importance. It presents the opposite of the totally independent case. Therefore, we shall call this kind of system a totally dependent system. The common output can be obtained from Eq. 3 by multiplying it with unity vector

$$DF^{-1}\Phi = DM \tag{23}$$

or in scalar form

$$\sum_k \mu_k = \phi \sum_k \frac{1}{F_k}. \tag{24}$$

The resultant transfer function for a totally dependent system according to Eq. 24 is obtained by summing the reciprocal transfer functions

$$\frac{1}{F} = \sum_k \frac{1}{F_k}. \tag{25}$$

If we take as an example that the subsystem transfer function consists of an n-order lag

$$F_k = \frac{1}{\sum a_k p^{n-k}} \tag{26}$$

we obtain for the independent system a rather complex resultant transfer function

$$F_i = \sum_j \frac{1}{\sum_k a_{jk} p^{n_j-k}}. \tag{27}$$

This function is in general of a different kind than the original transfer functions of the subsystems. For the totally dependent systems we obtain according to Eq. 25

$$F_d = \frac{1}{\sum A_k p^{n-k}}; \qquad A_k = \sum_j a_{kj}. \tag{28}$$

The resultant transfer function is of the same kind as the component functions, and the resultant parameters are simply the sums of all the corresponding parameters.

The results are somewhat paradoxical. The more strongly the subsystems are interrelated, the simpler the over-all behavior becomes. One may expect that this is approximately valid for the less strongly coupled systems. Therefore, there is no reason to expect that the performance of the system is more complex if the interrelations are discovered. On the contrary, if the interrelations are relatively strong, the over-all performance is simpler.

An important practical class of systems can be approximated by the totally dependent systems. If the interrelations are of integral type

$$V_{jk} = \frac{1}{p} W_{jk} \tag{29}$$

then condition 21 corresponds to the steady state of the interrelations. This case presents lossless media. If the compressibility of the media is not too high, the natural frequencies in the system due to the interrelations are much higher than the natural frequencies due to the subsystems. We can then assume, as an approximation, that the transient process due to the interrelations has ended much faster than the transient process due to the subsystems. We may then take the steady state of the interrelations, thus making the whole system totally dependent.

The mean value of ϕ can be of use in the analysis of the systems even if the natural frequencies of the interrelations and subsystems are closer to each other. If the interrelations between the neighboring subsystems are much stronger than between subsystems which are apart, we can first determine the mean mode ϕ by means of an approximation formula and thereafter determine the variable which is of particular interest

$$\phi_j = \phi + \left[\sum_{k\neq l} \prod_{k\neq j} V_{kl}\right]^{-1} \left[\sum \frac{1}{F_k}\right]^{-1} \left[\sum \frac{\mu_k}{F_k} + \mu_j \sum \frac{\prod(V_{kl} + V_{lm})}{F_k}\right]. \tag{30}$$

In the symmetric systems where $V_{jk} = V_{kj}$ we have

$$\phi_j = \phi + V^{-1} \left[\sum \frac{1}{F_k}\right]^{-1} \left[\sum_{k\neq j} \frac{\mu_k}{F_k} + 2\mu_j \sum_{k\neq j} \frac{1}{F_k}\right]. \tag{31}$$

This approximate formula can be used also for analogue representation. The whole system can be simulated to obtain the mean mode and additional simulation of Eq. 30 should be made only for the variable which is of particular interest.

We shall discuss here only one case more. If the hyperfluid matrix is of the form

$$V = \begin{pmatrix} 0, & \cdots\cdots\cdots & 0 \\ V_{21}, & 0, & \vdots \\ 0, & V_{32}, & \vdots \\ 0, & 0 \cdots V_{n,n-1}, & 0 \end{pmatrix} \tag{32}$$

or even of the form

$$V = \begin{bmatrix} 0, & 0, & 0, \cdots 0 \\ 1, & 0, & \vdots \\ 0, & 1, & \vdots \\ \cdot & \cdot & \vdots \\ 0, & 0, \cdots 1, & 0 \end{bmatrix} \tag{33}$$

all subsystems are connected in cascade in regard to the n^{th} output and first input. Paynter has shown (2) that if the transfer functions of all the subsystems are monotone, then the system as a whole will have some definite properties in the limit which did not exist in any of the subsystems if looked at separately.

From Eqs. 3 and 23 we obtain now in scalar form

$$\phi_n = F_1 F_2 \cdots F_n \mu_1. \tag{34}$$

Formalizing the output behavior, we obtain

$$\phi_{ns} = [\phi_n - \phi_{nm}]\phi_{n\infty}^{-1}. \tag{35}$$

In the limit, the mean mode of the normalized output is

$$T_m = \sum_k T_k \tag{36}$$

and the square deviation is

$$G = \sum_k G_k. \tag{37}$$

All higher moments are zero. The output variable is of Gaussian type. Quite independent of the actual kind of monotone transfer function of the subsystems, the normalized output variable is always the same and is of Gaussian type.

V. RELATIVE STRENGTH OF THE INTERRELATIONS

One important feature of large complex systems is their behavior when they consist of a very large number of subsystems (theoretically even of a denumerably infinite number). The characteristics of these systems depend upon the type of the transfer functions of the subsystems and upon the way they are involved in the system.

Three typical examples discussed in the previous section are: (1) The resultant transfer functions of the totally independent and totally dependent systems are obtained as the sums of the subsystems transfer functions or their inverse, respectively. The characteristics of every subsystem are preserved and can be recognized in the resultant transfer

function. (2) If the system consists of cascaded monotone subsystems, the behavior of the normalized output for very large systems is the same, regardless of the actual behavior of every subsystem (only they must be monotone). One of the important characteristics of the Gaussian output is its symmetry. Even if the asymmetry in the behavior of every subsystem is very strong (it can be of the same type for every subsystem), the output is still symmetric. This is valid theoretically only if there is an infinite number of subsystems, but in practice, a finite number (and sometimes even a few of them) also shows the same result. (3) As the representative of the third type, we can mention systems which are obtained by multiplying the outputs of the subsystems in the time domain. Even if there are only two subsystems, the features of the individual outputs can hardly be recognized.

The behavior of the system as a whole is determined by the properties of the subsystems and interrelations. How strong an influence the interrelating function has on the behavior of the outputs of the system determines the strength of the interrelations. Therefore, we can speak only about the strength of the interrelation in respect to the behavior of the subsystems themselves. Three typical examples defining relative strength of the interrelations are:

1. By definition, the interacting process is weaker than the functions of the subsystems behaviors if the characteristics of the subsystems can be fully recognized, even if the number of the subsystems is infinite. (Typical example of interacting process is summation.)
2. If ad limitum (and only ad limitum) the behavior of the system is dominated only by the type of the interacting process and not by the type of the functions of the subsystems behavior (providing that they all belong to a given class), the interacting process and functions of the subsystem behavior are of the same strength. (Typical example of interacting process is convolution.)
3. If the characteristics of the subsystems are hardly recognizable even after one (or a limited number) of applications of the interacting process, the functions of the subsystems behavior are by definition weaker than the interacting process. (Typical example of interacting function is multiplication.)

The above cases present extremity. In actual situations the relative strength of the interrelations lies between some of the extreme cases. When the hyperfluid is nonhomogeneous, the strength of the interrelations can be different in various parts of the system.

VI. FACTORS OF IMPORTANCE

The systems with interrelating functions of relatively high strength are often nonlinear in nature (Case 3) and therefore their detailed analysis is beyond the scope of this paper. Every linear system can be

represented as having addition and convolution as the only interrelating operations. The basic properties of convolution as an interacting operation are discussed in (10). We shall consider here some properties of addition as interacting operation.

In the preceding sections we have considered the finite sums of the infinite transfer functions of the subsystems. If the system has been subdivided into a great number of subsystems, the actual amplitudes of each component function should tend toward zero

$$n \to \infty ; \qquad f_j(t) \to 0. \tag{38}$$

The function $f_j(t)$ tends toward zero because of its minor significance in the ensemble of functions. However, if we look at every subsystem separately, we usually introduce the per unit parameters on the local basis.

A factor representing significance of the group of functions (with the same given parameter) in the system as a whole should now be introduced. We shall call it factor of importance and obtain

$$f_j(t) = k_j g_j(t). \tag{39}$$

For the whole system we have

$$\sum k_j = 1. \tag{40}$$

For the resultant transfer function in the finite case we obtain

$$f(t) = \sum_j k_j g_j(t). \tag{41}$$

If we have an infinite sum the factor of importance of each subsystem tends toward zero so that we have now

$$f(t) = \int_{-\infty}^{\infty} m_x g_{(x,t)} dx \tag{42}$$

where x—is a parameter characterizing a given class of the subsystems

$$m_x = \frac{dk_x}{dx}.$$

In general, the function $f(t)$ can have more parameters defining the class to which the function belongs so that we have

$$f(t) = \int_{\infty}^{\infty} da_n \cdots \int_{\infty}^{\infty} da_0 g_{(a_0,\cdots a_n)} m_{(a_0,\cdots a_n)}. \tag{43}$$

If the parameters change with time, the integral should be evaluated for every time instant. However, some of the subsystems can change their parameters with time, but still the integral can remain the same because we recognize the individual functions only by their belonging to a given class. If g_j represents the impulse response of the j^{th} subsystem the response of the system to a set of inputs is then

$$f(t) = \sum \sum k_j g_j(t) * \xi_i(t) \tag{44}$$

where the symbol * stays for convolution and $\xi_i(t)$ is the disturbance of the i^{th} kind acting on the j^{th} subsystem.

We shall consider here the case when more disturbances act on the system and when each of them affects every subsystem. Furthermore, we shall assume that the disturbances of the individual subsystems are of the same kind and, therefore, the distribution law of the disturbances affects only its relative amplitudes. Introducing the factor of importance for the disturbance, s_{ij} we obtain (Fig. 9)

$$f(t) = \sum \sum k_j g_j(t) * s_{ij} \zeta_i(t). \tag{45}$$

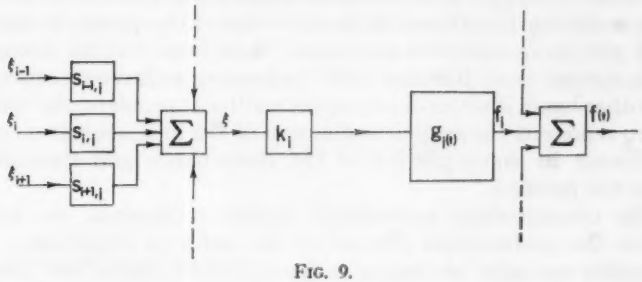

FIG. 9.

In the case of an infinite sum, we have

$$f(t) = \int_A \int_B m_{(a)} g_{(a,t)} * l_{(a,b)} \zeta_{(b,t)} da db \tag{46}$$

where b = parameter characterizing the distribution of the disturbance

$$l_{(a,b)} = \frac{\partial^2 s_{(a,b)}}{\partial a \partial b}$$

Very often the parameter b depends only upon the value of the parameter a in the moment when the disturbance occurs so that we have

$$f(t) = \int_A \int_{A_0} m_{(a)} g_{(a,t)} * l_{(a_0,a)} \zeta_{(a_0,t)} da_0 da. \tag{47}$$

The given method can be useful in the analysis of the systems composed by a summation procedure. We shall mention here only two examples.

It is well known that the frequency fluctuations in large electric power systems are much smaller in amplitudes than in small electric systems. The large inertia in the system and large operating power are often credited for this. However, if the characteristics of the electric networks are of the same kind, the way in which small systems are involved in a large one does not improve stability. The synchronizing torques can even produce additional troubles. The analysis by the given method shows, however, that the smaller frequency variations are not the consequence of greater stability in the structure of the system itself (on the contrary, a large system is less stable than a small one), but is due to the appropriate amplitude distribution of the disturbances ζ_j and of the factor of importance of the individual power plants and consumers.

Using the given method, the advantage of the "linearization by part" procedure over the linearization procedure using over-all behavior of the system can be shown.

If we have a large complex system consisting of the sum of the independent nonlinear functions, the linearization of the system is the more justified the more members we have. The best way to linearize a complex system is to linearize each and every subsystem separately. On the other hand, if we have a complex nonlinear problem, the best way is first to represent the system as the sum of the independent members, each affected by some portion of the disturbance and thereafter to linearize the problem.

If the characteristic parameters change at random, we have to introduce the probabilities instead of the factor of importance. The relationships are quite analogous to those derived above, but instead of k we have to introduce the probability distribution, and instead of m we have to introduce the frequency function, etc. The resultant function presents now the ensemble mean of the all possible functions for the given instant of time. If the variations of the parameters are stationary, the ensemble mean can be substituted with the time mean.

VII. DETERMINATION OF THE TRANSFER FUNCTIONS IN THE SYSTEM BY MEANS OF MEASUREMENTS

The problem of the experimental determination of the behavior of a totally dependent system is more complex than that of a totally independent system, and the methods applied are quite different. This is of principal importance because experimentation and measurement present the basis from which our knowledge of every physical system is derived. We shall demonstrate the difference at the problem of determining the transfer function between a given pair of input-output

in a linear multivariable system. Between the respective pairs of input-output, we can define a subsystem. There are two problems then: (1) to determine the transfer function of the subsystem, and (2) to determine the transfer function of the rest of the system. All other subsystems can be combined and an equivalent block diagram as shown in Fig. 11 is obtained.

In the case of a totally independent system F_{sm} is a zero matrix. Determination of the system transfer functions in the matrix F_m can be performed now only by measuring all the inputs and outputs.

In the strongly interrelated systems the matrix F_{sm} is dominant and F can be neglected (Fig. 10). The transfer function F_{sm} represents then the influence of the whole system on the subsystem under consideration. In general, the inputs can be distributed throughout the system so that we cannot separate the inputs from the reaction of the system. In our block diagram it means that we have no chance to put the instruments at points C and D. The transfer function of the system should be determined only by measuring the variables at points A and B. This fact presents the major challenge in the experimentation with the interrelated systems.

We shall consider the general case when the input functions belong to a random process. Consider the case when there are but one input and one output. The system is supposed to be linear, but we allow it to have time variable parameters. The relations between the variables are then

$$\phi(t) = \int_{-\infty}^{t} F_m(t, \tau)\mu_m(\tau)d\tau$$

$$\mu_m(t) = \mu(t) - \int_{-\infty}^{t} K_{sm}(t, \tau)\phi_{(\tau)}d\tau. \tag{53}$$

And around the whole loop

$$\mu(t) = \mu_m(t) + \int_{-\infty}^{t} d\tau_1 \int_{-\infty}^{t_1} d\tau_2 K_{sm}(t, \tau_1)F_m(t_1, \tau_2)\mu_m(\tau_2). \tag{54}$$

If the input is a random function, we can write the analogous relationships using correlation functions of the variables

$$\phi_{\varphi\varphi}(t_1, t_2) = \int_{-\infty}^{t_1} F_m(t_1, \tau_1)d\tau_1 \int_{-\infty}^{t_2} F_m(t_2, \tau_2)\phi_{\mu_m\mu_m}(\tau_1, \tau_2)d\tau_2 \tag{55}$$

$$\phi_{\mu\mu}(t_1, t_2) = \phi_{\mu_m\mu_m}(t_1, t_2) + \int_{-\infty}^{t_1} K_{sm}(t_1, \tau_1)d\tau_1 \int_{-\infty}^{t_2} K_{sm}(t_2, \tau_2) \times \phi_{\varphi\varphi}(\tau_1, \tau_2)d\tau_2 + \int_{-\infty}^{t_1} K_{sm}(t_1, \tau_1)\phi_{\varphi\mu_m}(t_2, \tau_1)d\tau_1 + \int_{-\infty}^{t_2} K_{sm}(t_2, \tau_2)\phi_{\mu_m\varphi}(t_1, \tau_2)d\tau_2. \tag{56}$$

The autocorrelations and cross correlations are taken over the ensemble of all possible values of the respective variables at the moments t_1 and t_2.

In Eq. 56 there are two unknowns: the weighting function K_{sm} and the correlation function of the input $\phi_{\mu\mu}$. One cannot measure the input directly, and therefore to eliminate one of the unknowns we should have one equation more. Because Eq. 56 fully describes our experiment (in terms of correlation functions) we have to carry out another experiment. To distinguish these two experiments, we have to change some of the properties of the system. One of the convenient ways for doing this is to change the state of the subsystem F_m, the input and output of which could be measured directly. In order to emphasize the essential ideas, we shall show the methodology of solution only for the stationary input and for the systems with constant parameters. One obtains then from Eqs. 55 and 56

$$\phi_{\varphi\varphi}(\tau) = \int_0^\infty F_m(\tau_1)d\tau_1 \int_0^\infty F_m(\tau_2)\phi_{\mu_m\mu_m}(\tau - \tau_1 + \tau_2)d\tau_2 \tag{57}$$

$$\phi_{\mu\mu}(\tau) = \phi_{\mu_m\mu_m(\tau)} + \int_0^\infty K_{sm}(\tau_1)d\tau_1 \int_0^\infty K_{sm}(\tau_2)\phi_{\varphi\varphi}(\tau - \tau_1 + \tau_2)d\tau_2 + \int_0^\infty K_{sm}(\tau_1)\phi_{\varphi\mu_m}(\tau + \tau_1)d\tau_1 + \int_0^\infty K_{sm}(\tau_2)\phi_{\mu_m\varphi}(\tau + \tau_2)d\tau_2 \tag{58}$$

where the autocorrelations are now defined as the time mean

$$\phi_{\mu\mu}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \mu(t)\mu(t + \tau)dt$$

$$\phi_{\varphi\varphi}(\tau) = \lim_{T\to\infty} \frac{1}{2T}\int_{-T}^{T} \phi(t)\phi(t + \tau)dt. \tag{59}$$

If we now change the state of the subsystem F_m and measure again the variables μ_m', and Φ', a new equilibrium equation analogous to Eq. 58 is obtained

$$\phi_{\mu'\mu'}(\tau) = \phi_{\mu_m'\mu_m'}(\tau) + \int_0^\infty K_{sm}(\tau_1)d\tau_1 \int_0^\infty K_{sm}(\tau_2) \times \phi_{\varphi'\varphi'2}(\tau - \tau_2 + \tau_1)d\tau_2 + \int_0^\infty K_{sm}(\tau_1)\phi_{\varphi'\mu_m'}(-\tau - \tau_1)d\tau_1 + \int_0^\infty K_{s..}(\tau_2)\phi_{\varphi'\mu_m'}(\tau - \tau_2)d\tau_2. \tag{60}$$

The variables μ_m' and Φ' are different from μ_m and Φ because of the randomness of the input functions. The autocorrelations $\phi_{\mu_m\mu_m}$ and

cross correlations $\phi_{\mu m}$ are also different because of the influence of the new state of the subsystem F_{sm}. But due to the stability of the input functions, the autocorrelations $\phi_{\mu\mu}$ and $\phi_{\mu'\mu'}$ are equal

$$\phi_{\mu\mu}(\tau) = \phi_{\mu'\mu'}(\tau). \tag{61}$$

Using this, we obtain from (58) and (61)

$$\begin{aligned}\phi_{\mu m\mu m}(\tau) - \phi_{\mu m'\mu m'}(\tau) \int_0^\infty K_{sm}(\tau_1)d\tau_1 \\ \times \int_0^\infty K_{sm}(\tau_2)[\phi_{\varphi\varphi}(\tau - \tau_2 + \tau_1) - \phi_{\varphi'\varphi'}(\tau - \tau_2 + \tau_1)]d\tau_2 \\ + \int_0^\infty K_{sm}(\tau_1)[\phi_{\varphi\mu m}(-\tau - \tau_1) - \phi_{\varphi'\mu m'}(-\tau - \tau_1)]d\tau_1 \\ + \int_0^\infty K_{sm}(\tau_2)[\phi_{\varphi\mu m}(\tau - \tau_2) - \phi_{\varphi'\mu m'}(\tau - \tau_2)]d\tau_2 = 0. \end{aligned} \tag{62}$$

The only unknown function in this equation is the weighting function of the system.

Equation 62 shows why we have written the relationships around the loop in a rather unusual form. We have to have an explicit expression for the unknown input function in order to avoid cross correlation functions between the unknown inputs and some other variables. The price that we had to pay for this is the complexity of Eq. 62. For a time invariable system Eq. 62 can be reduced to a convenient form by replacing the correlation functions with the spectral densities.

Application of the fourier transformation to both sides of Eq. 62 results in

$$\begin{aligned}G_{\mu m\mu m} - G_{\mu m'\mu m'} + \frac{1}{F_{sm}}[G_{\mu m\varphi} - G_{\mu m'\varphi'}] \\ + \frac{1}{\bar{F}_{sm}}[G_{\varphi\mu m} - G_{\varphi'\mu m'}] + \left|\frac{1}{F_{sm}}\right|^2 [G_{\varphi\varphi} - G_{\varphi'\varphi'}] = 0 \end{aligned} \tag{63}$$

where

$G_{\mu\mu}$, $G_{\mu'\mu'}$ = spectral density of the input
$G_{\varphi\varphi}$, $G_{\varphi'\varphi'}$ = spectral density of the output
$G_{\mu m\mu m}$ = spectral density of the local input
$G_{\mu m}$ = cross spectral density between the input and output
F_{sm} = transfer function of the system.

This equation should be solved for the unknown transfer function. However, in practice it often happens that the form of the transfer function is known from the previous physical analysis of the system. The task is then merely to determine the parameters for the case at hand.

Equation 63 can be written then in the convenient form

$$\alpha_{(\omega)}{}^2C_{(\omega)} + \alpha_{(\omega)}2A_{(\omega)} + T_{(\omega)}{}^2\omega^2C_{(\omega)} - T_{(\omega)}\omega^2 2B_{(\omega)} + D_{(\omega)} = 0 \quad (64)$$

where we have introduced the notations

$$\begin{aligned} G_{\varphi\mu m} - G_{\varphi'\mu m'} &= A_{(\omega)} + j\omega B_{(\omega)}; \qquad G_{\varphi\varphi} - G_{\varphi'\varphi'} = C_{(\omega)} \\ G_{\mu m\varphi} - G_{\mu m'\varphi'} &= A_{(\omega)} - j\omega B_{(\omega)}; \qquad G_{\mu m\mu m} - G_{\mu m'\mu m'} = D_{(\omega)} \\ \frac{1}{F_{sm}} &= \alpha_{(\omega)} + j\omega T_{(\omega)}. \end{aligned} \quad (65)$$

The real and imaginary parts of the unknown transfer function can now be obtained from the set of operation equations

$$\begin{aligned} &\alpha_{(\omega_1)}{}^2C_{(\omega_1)} + \alpha_{(\omega_1)}2A_{(\omega_1)} + T_{(\omega_1)}{}^2\omega_1{}^2C_{(\omega_1)} - T_{(\omega_1)}\omega_1{}^2 2B_{(\omega_1)} + D_{(\omega_1)} = 0 \\ &\qquad\vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \qquad\qquad\qquad \vdots \\ &\alpha_{(\omega_n)}{}^2C_{(\omega_n)} + \alpha_{(\omega_n)}2A_{(\omega_n)} + T_{(\omega_n)}{}^2\omega_n{}^2C_{(\omega_n)} - T_{(\omega_n)}\omega_n{}^2 2B_{(\omega_n)} + D_{(\omega_n)} = 0. \end{aligned} \quad (66)$$

This can easily be done by means of some computer. The number of equations in the set is equal to the number of unknown parameters.

After obtaining the unknown transfer function (or weighting function), we can determine the autocorrelation function or the spectral density of the input. This is usually very important because in such a way we have defined a resultant input which is valid for the whole system and therefore, the determination of the other transfer functions in the system can be significantly simplified.

In the application of the derived method, two further remarks are of importance: (1) It is not necessary to know in advance the transfer function (or weighting function) of the changed subsystem $F_{m'}$. We can determine it by measuring φ' and μ' and using the relation

$$\phi_{\varphi\mu m}(\tau) = \int_0^\infty F_m(t)\phi_{\mu m\mu m}(\tau - t)dt \quad (67)$$

or

$$|F_{m(j\omega)}|^2 = \frac{G_{\varphi m\varphi m}}{G_{\mu m\mu m}}. \quad (68)$$

(2) The change in the subsystem must be large enough to influence the whole system so that the output variable Φ could be altered. This is inevitable in order to avoid the identity in Eq. 64.

The same methodology can be applied to the more realistic but complicated case when the disturbance acts in the inside of the system itself

so the resultant transfer function of the system consists of the two unknown parts (Fig. 11). We have now to take three different states of the subsystems F_m, and using the outlined procedure, both of the functions can be obtained.

The equations can be analogously written for the linear system with the time varying parameters, but its solution will be more difficult.

The characteristic of the given method is that we did not know about the inputs anything more than that they belong to a stationary process. We did not know the probabilities or any statistical parameters. The only assumption was that the process is stationary. The fact that only by using such an assumption we succeed in determining a deterministic unknown function shows how restricted this assumption is.

If we want to use the similar method for the nonstationary inputs, we have to know the correlations over the ensemble of inputs for each moment or at least the relations between their correlation functions.

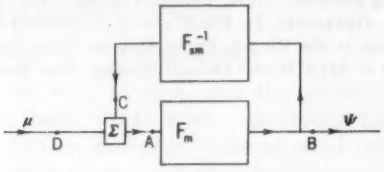

FIG. 10.

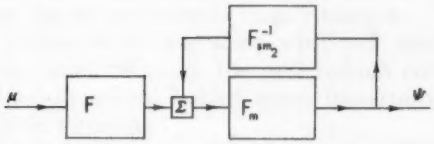

FIG. 11.

Acknowledgment

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THE ACCURACY OF IMPEDANCE MEASUREMENTS IN A.C. BRIDGES

BY

HANNS H. WOLFF [1]

ABSTRACT

The accuracy of impedance magnitude and phase measurements in A.C. bridges is derived as a function of the null instrument sensitivity and a bridge characteristic.

The influence of the bridge-length ratio and its phase angle on the measurement accuracy is treated in detail. It is especially shown that for a bridge-length ratio amount of 1, that is, $|z_1| = |z_3|$, the obtainable accuracy improves as the phase angle $\sphericalangle z_1, z_3$ increases.

It is furthermore shown that the maximum impedance magnitude error and the maximum phase error occur at different bridge adjustments. Formulas for the phase angles between null voltage and bridge voltage for maximum impedance magnitude error and for maximum phase error are derived.

1. INTRODUCTION

The sensitivity of A.C. bridges and the accuracy of A.C. bridge measurements have been dealt with repeatedly (see Bibliography). Most of the prior studies, however, concentrated on the bridge sensitivity and assumed that a more sensitive bridge automatically leads to a more accurate measurement. Furthermore, the sharper the "Null" appeared, the better a bridge adjustment was considered to be. This study attempts to clarify and correct these concepts.

Most of the former work was done with null instruments whose impedance was too small to make the null branch current negligible. Theoretical studies, therefore, buried some important facts under a bulk of impenetrable formulas.

Seletzky and Zurcher[2] broke with the past and have given us a deeper insight into modern A.C. bridges by limiting their studies to bridges with negligible null branch currents. The present paper follows this approach. However, instead of being concerned with the "sensitivity of the bridge" as such, it emphasizes the accuracy of the measurement directly as a function of the bridge characteristics and the null instrument sensitivity. This paper deals exclusively with the error which is due to a limited null instrument sensitivity. Systematic errors due to errors in the assumed data of the bridge parameters and stray fields have to be taken into consideration additionally.

It is often assumed that an increase in null instrument sensitivity

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[2] A. C. Seletzky and L. A. Zurcher, "Sensitivity of the Four-Arm Bridge," *AIEE Trans.*, Vol. 58, pp. 723–28 (1939).

improves the obtainable accuracy. However, if the sensitivity is driven to the technically possible limit, stray fields and other unavoidable influences often offset the advantages of a higher sensitivity by misleading the operator and by increasing layout and operational difficulties.

2. NOMENCLATURE

Figure 1 shows the A.C. bridge circuit with definition of voltages, currents and impedances. Voltages, currents and impedances are given in phasor notation.

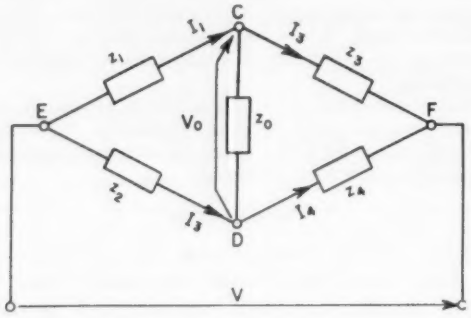

FIG. 1. Bridge—nomenclature.

V = bridge supply voltage
V_0 = null instrument (null branch) voltage
φ_0 = phase angle between null instrument voltage and bridge supply voltage
$|V_s|$ = voltage sensitivity of null instrument
z_1, z_2, z_3, z_4 = bridge arm impedances (z_2 is assumed to be the unknown impedance, z_4 the variable impedance)
$\varphi_1, \varphi_2, \varphi_3, \varphi_4$ = phase angles of bridge-arm impedances
z_0 = null instrument impedance
I_1, I_2, I_3, I_4 = bridge-arm currents
$k = z_1/z_3$ = constant series impedance ratio (z_1 and z_3 are assumed to be fixed impedances)
$\varphi_k = \varphi_1 - \varphi_3$ = constant series impedance ratio phase angle
$\varphi_{24} = \varphi_2 - \varphi_4$ = variable series impedance ratio phase angle
$|\Delta z_2|$ = maximum measurement error of impedance magnitude $|z_2|$ due to the limitation in null instrument sensitivity
$\Delta\varphi_2$ = maximum measurement error of phase angle φ_2 of impedance z_2 due to the limitation in null instrument sensitivity.

E, F = bridge supply nodes
C, D = bridge null nodes
A, B, g, P, Q, ψ = substitute terms (see Eqs. A3, A4, 14, C8, C9 and C11.

The index "max/min" is used to indicate that the upper sign belongs to the maximum, the lower to the minimum value. The index "min/max" is used analogously. The index "max min" leaves the sign assignment open.

φ_{0+} is the amount of φ_0 for the positive, φ_{0-} for the negative sign in Eqs. 29 and 27.

(For equations designated (A..), (B..) or (C..) see Appendices.)

3. BASIC EQUATIONS

As mentioned in the introduction it shall be assumed that

$$|z_0| \gg |z_1|, |z_2|, |z_3| \quad \text{and} \quad |z_4| \tag{1}$$

such that for all purposes of this study

$$I_1 = I_3 \tag{2}$$
$$I_2 = I_4. \tag{3}$$

Then follows, from Fig. 1,

$$I_1 z_1 - V_0 = I_2 z_2 \tag{4}$$

$$I_1 z_3 + V_0 = I_2 z_4 \tag{5}$$

$$I_1 = V/(z_1 + z_3). \tag{6}$$

Dividing Eq. 5 into Eq. 4 and inserting Eq. 6 results in

$$\begin{aligned} \frac{z_2}{z_4} &= \frac{I_1 z_1 - V_0}{I_1 z_3 + V_0} \\ &= \frac{\dfrac{z_1}{z_3} - \dfrac{V_0}{I_1 z_3}}{1 + \dfrac{V_0}{I_1 z_3}} \\ &= \frac{\dfrac{z_1}{z_3} - \dfrac{V_0}{V}\left(\dfrac{z_1}{z_3} + 1\right)}{1 + \dfrac{V_0}{V}\left(\dfrac{z_1}{z_3} + 1\right)}. \end{aligned} \tag{7}$$

It shall be assumed for this study that the bridge is so adjusted that

$$|V_0| \ll |V| \tag{8}$$

and

$$\left|\frac{V_0}{V}\frac{z_1}{z_3}\right| \ll 1. \tag{9}$$

Then, from Eq. 7,

$$\begin{aligned}\frac{z_2}{z_4} &\approx \left[\frac{z_1}{z_3} - \frac{V_0}{V}\left(\frac{z_1}{z_3}+1\right)\right]\left[1 - \frac{V_0}{V}\left(\frac{z_1}{z_3}+1\right)\right] \\ &\approx \frac{z_1}{z_3} - \frac{V_0}{V}\left(\frac{z_1}{z_3}+1\right)^2 \\ &= k - \frac{V_0}{V}(k+1)^2 \end{aligned} \tag{10}$$

from which $|z_2/z_4|$ can be derived as a function of $|V_0/V|$, φ_0, $|k|$ and φ_k. (For details see Appendix A.)

4. THE MAXIMUM ERROR IN IMPEDANCE MAGNITUDE MEASUREMENT

Figure 2 shows the phasor diagram of an unbalanced bridge. A

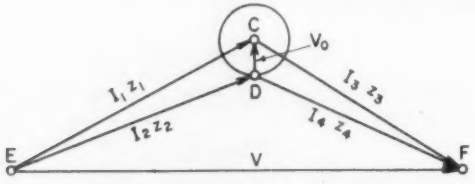

FIG. 2. Bridge—phasor diagram.

given magnitude of null branch voltage $|V_0|$ can obviously be caused by any z_2/z_4 adjustment whose locus for the node point D (see Fig. 1) is the circle with center at C.

If the radius of the circle in the diagram is chosen such that it corresponds to the null instrument voltage sensitivity $|V_s|$, the bridge appears to be balanced if, by adjustment of z_2/z_4, the node point D is forced anywhere onto or inside the circle. The circle area is, therefore, the locus for all adjustments for which the bridge appears to be balanced. The maximum error will be caused by certain z_2/z_4 adjustments, for which the node point is somewhere on the circle.

For a given $k = z_1/z_3$ and a given $|V_0/V|$, the maximum error of $|z_2/z_4|$, and thereby of $|z_2|$, can therefore be found by calculating the maximum and minimum values of $|z_2/z_4|$. By selecting $|V_0| = |V_s|$

(the maximum voltage which can no longer be distinguished from zero), it is then possible to predict the maximum error in the evaluation of the magnitude of the unknown impedance which is due to the limitation in the null instrument sensitivity.

From the maximum-minimum relation

$$\frac{d}{d\varphi_0}\left(\left|\frac{z_2}{z_4}\right|\right) = 0 \tag{11}$$

it follows that

$$\left|\frac{z_2}{z_4}\right|_{\max/\min} = |k|\left[1 \pm \left|\frac{V_s}{V}\right|\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)\right]. \tag{12}$$

(For details see Appendix B.) The positive sign in Eq. 12 yields $|z_2/z_4|_{\max}$, the negative one $|z_2/z_4|_{\min}$.

As mentioned above, it is assumed that z_4 is the variable bridge parameter and therefore

$$|z_2/z_4|_{\max} = |z_2|/|z_4|_{\min} \quad \text{and} \quad |z_2/z_4|_{\min} = |z_2|/|z_4|_{\max}.$$

The absolute error $|\Delta z_2|$ is that amount by which the result of the test is higher than the true value of $|z_2|$

$$\frac{|z_2| + |\Delta z_2|}{|z_4|_{\max/\min}} = |k|. \tag{13}$$

($|z_4|_{\min}$ will result in a negative value for $|\Delta z_2|$). If

$$g = + |k|\left|\frac{V_s}{V}\right|\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right) \tag{14}$$

is introduced, it follows from Eq. 12 that

$$\begin{aligned} |z_2| &= |z_4|_{\min/\max}(|k| \pm g) \\ &= |z_4|_{\max/\min}(|k| \mp g). \end{aligned} \tag{15}$$

Inserting Eq. 15 into Eq. 13 gives

$$|\Delta z_2| = |z_4|_{\max/\min}(\pm g) \tag{16}$$

or the relative error

$$\frac{|\Delta z_2|}{|z_2|} = \frac{|z_4|_{\max/\min}}{|z_2|} \cdot (\pm g)$$

(Continued on next page)

$$\approx \frac{1}{|k|} \cdot (\pm g)$$

$$= \pm \left| \frac{V_e}{V} \right| \left(|k| + \frac{1}{|k|} + 2 \cos \varphi_k \right). \tag{17}$$

Equation 17 shows that the relative error $|\Delta z_2|/|z_2|$ is proportional to the null instrument sensitivity ratio $|V_e|/|V|$ and a bridge characteristic $|k| + 1/|k| + 2 \cos \varphi_k$. For a given φ_k, this bridge characteristic becomes larger as the series bridge ratio deviates farther from 1, though small changes in $|k|$ do not influence the relative error noticeably. However, at least for values of $|k|$ close to 1, the phase angle φ_k of the series bridge ratio can influence the error noticeably. The function $|k| + 1/|k| + 2 \cos \varphi_k$ is shown in Fig. 3.

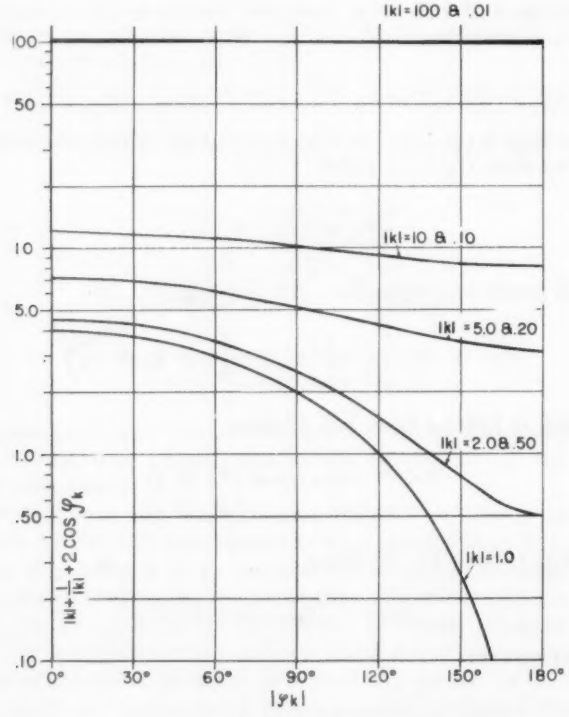

FIG. 3. The bridge characteristic $|k| + \frac{1}{|k|} + 2 \cos \varphi_k$ as a function of the series impedance ratio phase angle φ_k for several values of the bridge-length ratio amount $|k|$ as parameter.

It is evident from this figure that it is possible to increase the obtainable accuracy for a given $|k|$ by selecting a favorable φ_k, at least if $|k|$ deviates only slightly from the value 1. It should furthermore be noted that the maximum relative error in the measurement of the magnitude of the impedance is independent of the sign of φ_k.

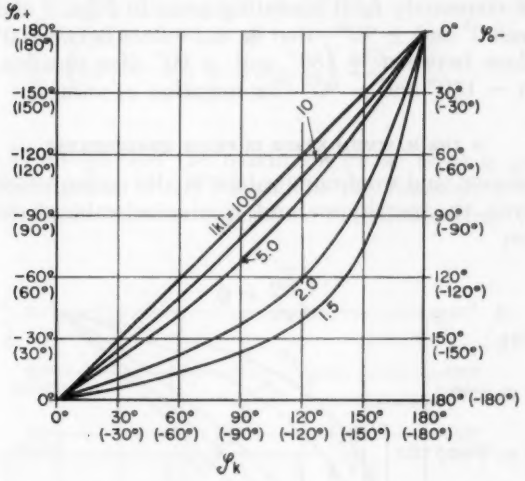

FIG. 4. The null branch *versus* supply voltage phase angle φ_0 for maximum impedance error as a function of the series impedance ratio phase angle φ_k for several values of the bridge-length ratio amount $|k|$ as parameter.

It is of scientific more than of practical interest to know under which φ_0 the maximum relative error in impedance magnitude measurement occurs. Equations A5 and A7 substituted into Eq. B3 give

$$\cos \varphi_0 = \pm \frac{\left(|k| + \frac{1}{|k|}\right) \cos \varphi_k + 2}{|k| + \frac{1}{|k|} + 2 \cos \varphi_k}. \tag{18}$$

Equation 18 shows that the phase angle φ_0 between null instrument voltage and bridge voltage under which the maximum error occurs is independent of φ_k when $|k| = 1$, and independent of $|k|$ when $\varphi_k = 0$. In both cases, it follows from Eq. 18 that

$$\cos \varphi_0 = \pm 1, \tag{19}$$

that is,

$$\varphi_0 = 0°, \quad \pm 180°. \tag{20}$$

Figure 4 shows φ_{0+} and φ_{0-} as a function of φ_k for several values of $|k|$. φ_{0+} is the amount of φ_0 for the positive, φ_{0-} for the negative sign in Eq. 18. The values in parentheses refer to negative φ_k angles. To each $|k|$, φ_k combination belong one φ_{0+} and one φ_{0-} value.

If the graphs shall be used for the reciprocals of the indicated $|k|$ values, the sign of the φ_{0+} and φ_{0-} angles has to be changed.

The most commonly used operating areas in Figs. 3 and 4 cover φ_k values between 0° and ± 90°—that is, φ_{0+} values between 0° and ∓ 90° and φ_{0-} values between + 180° and + 90° (for positive φ_k values) and between − 180° and − 90° (for negative φ_k values).

5. THE MAXIMUM ERROR IN PHASE MEASUREMENT

The maximum and minimum values in the measurement of φ_2 can be found from the maximum and minimum values which φ_{24} can assume. From

$$\frac{d\varphi_{24}}{d\varphi_0} = 0, \tag{21}$$

it follows that

$$\varphi_{24_{\min/\max}} = \varphi_2 - \varphi_{4_{\max/\min}}$$

$$= \varphi_k \mp \arctan\left[\left|\frac{V_s}{V}\right| \frac{1}{\sqrt{\frac{1}{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2} - \left|\frac{V_s}{V}\right|^2}}\right] \tag{22}$$

(For details see Appendix C.)

Since we calculate $\varphi_2 + \Delta\varphi_2$ instead of φ_2 from φ_4 and φ_k, that is,

$$\varphi_2 + \Delta\varphi_2 = \varphi_k + \varphi_{4_{\max/\min}}, \tag{23}$$

it follows from Eqs. 22 and 23 that

$$\Delta\varphi_2 = \pm \arctan\left[\left|\frac{V_s}{V}\right| \frac{1}{\sqrt{\frac{1}{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2} - \left|\frac{V_s}{V}\right|^2}}\right] \tag{24}$$

and, with

$$\left|\frac{V_s}{V}\right| \ll \frac{1}{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)}, \tag{25}$$

the phase error becomes

$$\Delta\varphi_2 \approx \pm \left[\left| \frac{V_s}{V} \right| \frac{1}{\sqrt{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k \right)^2} - \left| \frac{V_s}{V} \right|^2} \cdot \frac{180}{\pi} \right]^\circ$$

$$\approx \pm \left| \frac{V_s}{V} \right| \left(|k| + \frac{1}{|k|} + 2\cos\varphi_k \right) \frac{180}{\pi}{}^\circ. \tag{26}$$

Equation 26 shows that the maximum phase error is proportional to the sensitivity ratio $|V_s|/|V|$ and to the function $(|k| + 1/|k| + 2\cos\varphi_k)$, as is the relative impedance magnitude error (see Eq. 17 and Fig. 3).

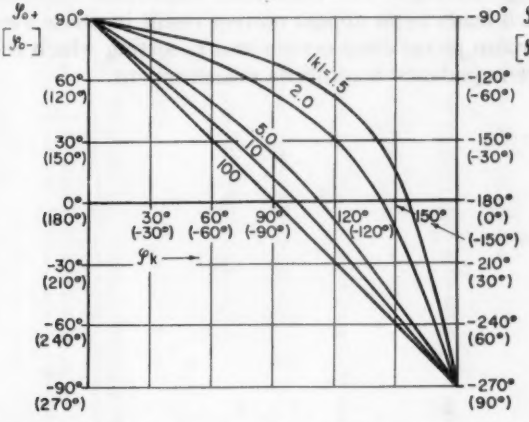

FIG. 5. The null branch *versus* supply voltage phase angle φ_0 for maximum phase error as a function of the series impedance ratio phase angle φ_k for several values of the bridge-length ratio amount $|k|$ as parameter.

For the phase angle φ_0 under which the maximum phase error $\Delta\varphi_2$ occurs, one obtains (see Eqs. C4, A5, C7 and 25)

$$\varphi_0 = \arccos \left\{ \left| \frac{V_s}{V} \right| \left[\left(|k| + \frac{1}{|k|} \right) \cos\varphi_k + 2 \right] \pm \frac{\sin\varphi_k \left(|k| - \frac{1}{|k|} \right)}{|k| + \frac{1}{|k|} + 2\cos\varphi_k} \right\}. \tag{27}$$

If the term containing $|V_s|/|V|$ is very small, then, from Eq. 27,

$$\varphi_0 \approx \pm 90° \tag{28}$$

both for $\varphi_k = 0$ and for $|k| = 1$.

Figure 5 shows the change in φ_{0+} and φ_{0-} with φ_k as a variable for several values of $|k|$, provided the term with $|V_s/V|$ can be ignored. The values in parentheses refer to negative φ_k angles.

If the graphs are to be used for the reciprocal of the indicated parameter $|k|$ as given in the figure, the bracketed $[\varphi_{0+}]$ and $[\varphi_{0-}]$ scales must be used and the signs of the $[\varphi_{0+}]$ and $[\varphi_{0-}]$ values have to be changed.

To avoid mistakes it is always advisable to derive the sign and value range of φ_0 from a phasor diagram.

A comparison of Figs. 4 and 5 or a phasor diagram shows that maximum impedance error and maximum phase error do not occur at the same bridge setting. The maximum impedance error occurs at a z_4 setting which leads to an almost correct result in phase measurement and the maximum phase error occurs at a z_4 setting which results in an almost correct impedance magnitude measurement.

APPENDIX A

From Eq. 10, one obtains

$$\left|\frac{z_2}{z_4}\right| e^{j(\varphi_2-\varphi_4)} = |k|e^{j\varphi_k} - \left|\frac{V_0}{V}\right| e^{j\varphi_0}(|k|e^{j\varphi_k}+1)^2$$

$$= |k|e^{j\varphi_k} - \left|\frac{V_0}{V}\right| e^{j\varphi_0}(|k|^2e^{j2\varphi_k}+2|k|e^{j\varphi_k}+1) \tag{A1}$$

and, from Eq. A1,

$$\left|\frac{z_2}{z_4}\right| = \sqrt{\left[|k|\cos\varphi_k - \left|\frac{V_0}{V}\right| |k|^2\cos(\varphi_0+2\varphi_k) - 2\left|\frac{V_0}{V}\right| |k|\cos(\varphi_0+\varphi_k) - \left|\frac{V_0}{V}\right| \cos\varphi_0\right]^2 + \left[|k|\sin\varphi_k - \left|\frac{V_0}{V}\right| |k|^2\sin(\varphi_0+2\varphi_k) - 2\left|\frac{V_0}{V}\right| |k|\sin(\varphi_0+\varphi_k) - \left|\frac{V_0}{V}\right| \sin\varphi_0\right]^2}$$

$$\left|\frac{z_2}{z_4}\right| = \sqrt{\begin{array}{l}\left[|k|\cos\varphi_k+\cos\varphi_0\left|\frac{V_0}{V}\right| (-|k|^2\cos 2\varphi_k-2|k|\cos\varphi_k-1)+\sin\varphi_0\left|\frac{V_0}{V}\right| (|k|^2\sin 2\varphi_k+2|k|\sin\varphi_k)\right]^2 \\ +\left[|k|\sin\varphi_k+\cos\varphi_0\left|\frac{V_0}{V}\right| (-|k|^2\sin 2\varphi_k-2|k|\sin\varphi_k)+\sin\varphi_0\left|\frac{V_0}{V}\right| (-|k|^2\cos 2\varphi_k-2|k|\cos\varphi_k-1)\right]^2\end{array}}. \tag{A2}$$

Setting

$$-\left|\frac{V_0}{V}\right| (|k|^2\cos 2\varphi_k + 2|k|\cos\varphi_k + 1) = A, \tag{A3}$$

$$+\left|\frac{V_0}{V}\right| (|k|^2\sin 2\varphi_k + 2|k|\sin\varphi_k) = B \tag{A4}$$

one obtains

$$A\cos\varphi_k - B\sin\varphi_k = -\left|\frac{V_0}{V}\right| (|k|^2\cos\varphi_k + 2|k| + \cos\varphi_k)$$

$$= -\left|\frac{V_0}{V}\right| |k| \left[\left(|k| + \frac{1}{|k|}\right)\cos\varphi_k + 2\right], \tag{A5}$$

$$B\cos\varphi_k + A\sin\varphi_k = \left|\frac{V_0}{V}\right| (|k|^2\sin\varphi_k - \sin\varphi_k)$$

$$= \left|\frac{V_0}{V}\right| |k| \left(|k| - \frac{1}{|k|}\right)\sin\varphi_k \tag{A6}$$

and

$$A^2 + B^2 = \left|\frac{V_0}{V}\right|^2 (|k|^4 + 4|k|^3 + 1 + 4|k|^3\cos\varphi_k + 2|k|^2\cos 2\varphi_k + 4|k|\cos\varphi_k)$$

$$= \left|\frac{V_0}{V}\right|^2 [(|k|^2+1)^2 + 4|k|(|k|^2+1)\cos\varphi_k + 4|k|^2\cos^2\varphi_k] \tag{A7}$$

$$= \left|\frac{V_0}{V}\right|^2 (|k|^2 + 1 + 2|k|\cos\varphi_k)^2,$$

which will be needed later.

Then, from Eqs. A2, A3 and A4,

$$\left|\frac{z_2}{z_4}\right| = \sqrt{(|k| \cos \varphi_k + A \cos \varphi_0 + B \sin \varphi_0)^2 + (|k| \sin \varphi_k - B \cos \varphi_0 + A \sin \varphi_0)^2}$$

$$= \sqrt{|k|^2 + A^2 + B^2 + 2|k| \cos \varphi_0 (A \cos \varphi_k - B \sin \varphi_k) + 2|k| \sin \varphi_0 (B \cos \varphi_k + A \sin \varphi_k)} \tag{A8}$$

APPENDIX B

Applying Eq. 11 to Eq. A8, one obtains

$$-\sin \varphi_0 (A \cos \varphi_k - B \sin \varphi_k) + \cos \varphi_0 (B \cos \varphi_k + A \sin \varphi_k) = 0 \tag{B1}$$

which reduces to

$$\sin \varphi_0 = \cos \varphi_0 \frac{B \cos \varphi_k + A \sin \varphi_k}{A \cos \varphi_k - B \sin \varphi_k}. \tag{B2}$$

Squaring Eq. B2 yields

$$\cos \varphi_0 = \frac{A \cos \varphi_k - B \sin \varphi_k}{\pm \sqrt{A^2 + B^2}} \tag{B3}$$

and substitution of Eqs. B2 and B3 into Eq. A8 yields with Eq. A7 finally, Eq. 12.

APPENDIX C

From Eqs. A1 through A4, one obtains

$$\varphi_2 - \varphi_4 = \varphi_{24}$$

$$= \arctan \frac{|k| \sin \varphi_k - B \cos \varphi_0 + A \sin \varphi_0}{|k| \cos \varphi_k + A \cos \varphi_0 + B \sin \varphi_0} \tag{C1}$$

which, when inserted into Eq. 21, yields

$$(B \sin \varphi_0 + A \cos \varphi_0)(|k| \cos \varphi_k + A \cos \varphi_0 + B \sin \varphi_0) - (-A \sin \varphi_0 + B \cos \varphi_0)(|k| \sin \varphi_k - B \cos \varphi_0 + A \sin \varphi_0) = 0. \tag{C2}$$

From Eq. C2, one obtains

$$\sin \varphi_0 = \frac{-\frac{A^2 + B^2}{|k|} - \cos \varphi_0 (A \cos \varphi_k - B \sin \varphi_k)}{B \cos \varphi_k + A \sin \varphi_k} \tag{C3}$$

and, from Eq. C3

$$\cos \varphi_0 = -\frac{A \cos \varphi_k - B \sin \varphi_k}{|k|} \pm \sqrt{\frac{(A \cos \varphi_k - B \sin \varphi_k)^2}{|k|^2} - \frac{A^2 + B^2}{|k|^2} + \frac{(B \cos \varphi_k + A \sin \varphi_k)^2}{A^2 + B^2}}. \tag{C4}$$

Substitution of Eqs. C3 and C4 into C1 gives

$$\varphi_{2\text{max min}} = \arctan \frac{|k| \sin\varphi_k (B\cos\varphi_k + A\sin\varphi_k) - \frac{A}{|k|}(A^2+B^2) + [-\cos\varphi_k (A^2+B^2)]\left(-\frac{A\cos\varphi_k - B\sin\varphi_k}{|k|} \pm \sqrt{\frac{(A\cos\varphi_k - B\sin\varphi_k)^2}{|k|^2} - \frac{A^2+B^2}{|k|^2} + \frac{(B\cos\varphi_k + A\sin\varphi_k)^2}{A^2+B^2}}\right)}{|k| \cos\varphi_k (B\cos\varphi_k + A\sin\varphi_k) - \frac{B}{|k|}(A^2+B^2) + [\sin\varphi_k (A^2+B^2)]\left(-\frac{A\cos\varphi_k - B\sin\varphi_k}{|k|} \pm \sqrt{\frac{(A\cos\varphi_k - B\sin\varphi_k)^2}{|k|^2} - \frac{A^2+B^2}{|k|^2} + \frac{(B\cos\varphi_k + A\sin\varphi_k)^2}{A^2+B^2}}\right)}$$

$$= \arctan \frac{\tan\varphi_k (B\cos\varphi_k + A\sin\varphi_k)\left(\frac{|k|}{A^2+B^2} - \frac{1}{|k|}\right) - \left(\pm\sqrt{\frac{(A\cos\varphi_k - B\sin\varphi_k)^2}{|k|^2} - \frac{A^2+B^2}{|k|^2} + \frac{(B\cos\varphi_k + A\sin\varphi_k)^2}{A^2+B^2}}\right)}{(B\cos\varphi_k + A\sin\varphi_k)\left(\frac{|k|}{A^2+B^2} - \frac{1}{|k|}\right) + \tan\varphi_k \left(\pm\sqrt{\frac{(A\cos\varphi_k - B\sin\varphi_k)^2}{|k|^2} - \frac{A^2+B^2}{|k|^2} + \frac{(B\cos\varphi_k + A\sin\varphi_k)^2}{A^2+B^2}}\right)}. \tag{C5}$$

From Eqs. A6 and A7, one obtains

$$(B\cos\varphi_k + A\sin\varphi_k)\left(\frac{|k|}{A^2+B^2} - \frac{1}{|k|}\right) = \sin\varphi_k \left(|k| - \frac{1}{|k|}\right)\left[\frac{1}{\left|\frac{V_0}{V}\right|\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2} - \left|\frac{V_0}{V}\right|\right] \tag{C6}$$

and, from Eqs. A5, A6 and A7

$$\pm\sqrt{\frac{(A\cos\varphi_k - B\sin\varphi_k)^2}{|k|^2} - \frac{A^2+B^2}{|k|^2} + \frac{(B\cos\varphi_k + A\sin\varphi_k)^2}{A^2+B^2}}$$

$$= \pm\sqrt{\left|\frac{V_0}{V}\right|^2\left[\left(|k| + \frac{1}{|k|}\right)^2(-\sin^2\varphi_k) + 4\sin^2\varphi_k\right] + \left[\frac{\left(|k| - \frac{1}{|k|}\right)\sin\varphi_k}{|k| + \frac{1}{|k|} + 2\cos\varphi_k}\right]^2}$$

$$= \pm\sin\varphi_k \left(|k| - \frac{1}{|k|}\right)\sqrt{-\left|\frac{V_0}{V}\right|^2 + \frac{1}{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2}}. \tag{C7}$$

By setting

$$\frac{1}{\left|\frac{V_0}{V}\right|\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2} - \left|\frac{V_0}{V}\right| = P \tag{C8}$$

and

$$\sqrt{\frac{1}{\left(|k| + \frac{1}{|k|} + 2\cos\varphi_k\right)^2} - \left|\frac{V_0}{V}\right|^2} = Q \tag{C9}$$

and substituting Eqs. C6, C7, C8 and C9 into Eq. C5, one obtains

$$\varphi_{24_{\max\min}} = \arctan \frac{P \tan \varphi_k - (\pm Q)}{P + (\pm Q) \tan \varphi_k}$$

$$= \arctan \frac{\tan \varphi_k \mp \frac{Q}{P}}{1 \pm \frac{Q}{P} \tan \varphi_k}. \tag{C10}$$

By setting

$$+\frac{Q}{P} = \tan \psi, \tag{C11}$$

and substituting in Eq. C10, one obtains

$$\varphi_{24_{\max\min}} = \arctan \frac{\tan \varphi_k \mp \tan \psi}{1 \pm \tan \varphi_k \tan \psi}$$

$$= \arctan \tan(\varphi_k \mp \psi) \tag{C12}$$

which yields

$$\varphi_{24_{\min/\max}} = \varphi_k \mp \psi$$

$$= \varphi_k \mp \arctan\left(+\frac{Q}{P}\right). \tag{C13}$$

In calculating the maximum phase error due to the limitation in null instrument sensitivity $|V_s|$, if $|V_s| = |V_0|$ is introduced and Eqs. C8 and C9 are substituted into Eq. C13, Eq. 22 results.

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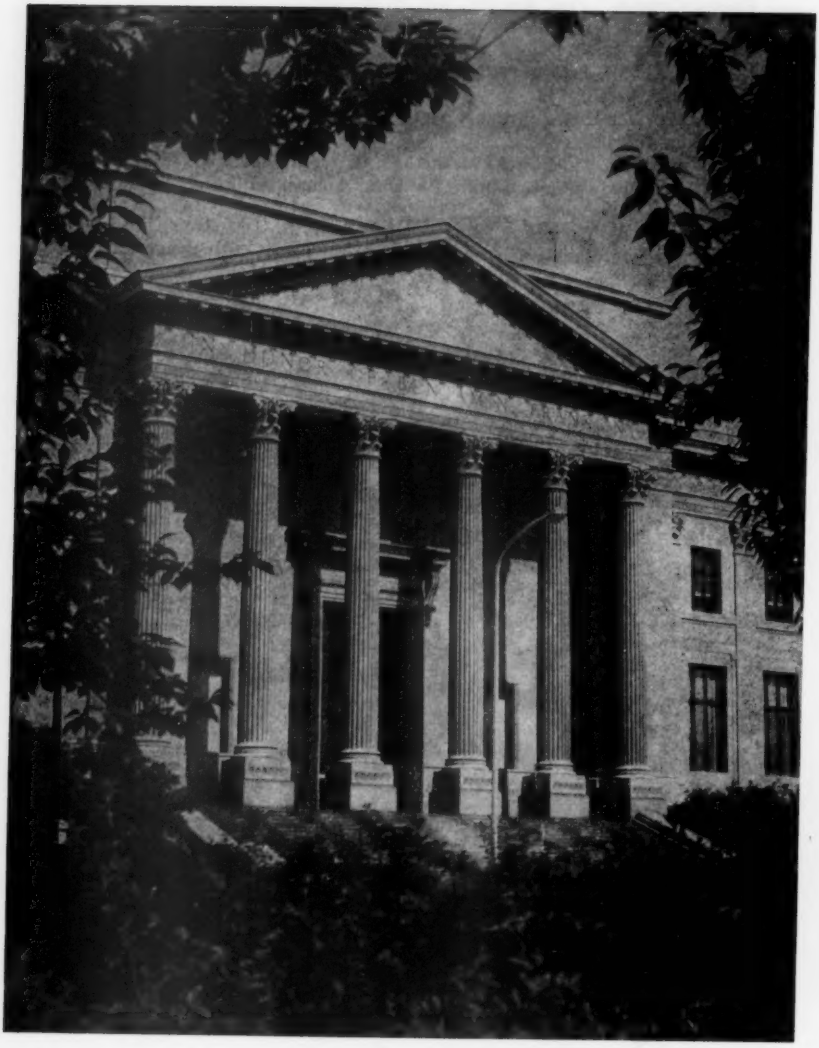


The Franklin Institute
ANNUAL REPORT

of the Board of Managers for the Year 1959

Summary

The year 1959 marked the 135th anniversary of the founding of The Franklin Institute and the 25th year of its fine edifice on the Parkway in Philadelphia. In presenting the Annual Report for 1959, the Board of Managers notes that important progress has been made in fulfilling the promise of the organization founded so modestly in 1824 for the mutual benefit of a few citizens of Philadelphia. In contradistincton, the sphere of influence of The Franklin Institute in 1959 extended from Pole to Pole, from West to East around the World, and far out into Space.

The founding fathers, whose principal aim was the promotion of the mechanic arts among themselves, would be amazed at the present scope of the Institute's science promotion activities, which in 135 years has grown to global proportions, covering a wide range, from young to old in age, from beginners to professionals in experience, and from teaching to research in method.

In this Report we present the highlights of 1959 in the form of a spiral, the focal point of which is The Franklin Institute in Philadelphia. The unbroken and ever-widening curves of the spiral reach out through the Delaware Valley, the Nation, the World, and the Universe beyond our planet.

Since our activities originate at the Institute, Delaware Valley naturally feels the impact of all of them. This Report, however, deals with each activity within the geographical area for which its contributions to science are the largest. For example, our Laboratories, though performing valuable research for local firms, are dealt with under "The Nation," because a greater volume of their work is done for the Government.

Residents of Philadelphia and Delaware Valley are very nearly exclusive beneficiaries of five of the Institute's activities: the Science Museum, the Astronomy Department, Educational Radio and Television, the Meetings program, and the Library. To take advantage of the opportunities offered through these activities, those wishing to learn must, in general, come to the Institute's headquarters where the facilities are located.

During 1959, the Science Museum was visited by 410,433 people, of whom approximately 290,000 were school children. Regular features such as the Workshops, the Science Fairs, the Dodge Lecture, and the Career Forums all enjoyed marked gains in attendance over previous years. Of the sixteen per-

manent exhibits added during the year the most spectacular are the full-size "Matador" missile located outside on the southeast terrace, and the Smoke Tunnel, to be seen in the Hall of Aviation. Guided tours of the Laboratories for high school students were initiated and were so successful that they will be made a permanent part of the program. By far the most significant innovation was the inauguration in November of regular classes during school hours. Arranged with the cooperation of the Board of Education of the School District of Philadelphia, public school classes now come to the Museum to study different phases of science, under a certified teacher. These classes utilize various exhibits to supplement their regular class work. We have great expectations for the continuation and expansion of this activity.

The Astronomy Department devoted six months in the Planetarium to space travel. Excellent timing for the Moon show assured record attendance. The Department obtained several unique exhibits for loan display, including a Vanguard Exhibit and a model of the 600-ft. radiotelescope under construction in West Virginia. It is of interest to note that the Institute's Laboratories worked on the bearing designs for this great radiotelescope.

In the relatively new field of Educational Radio and Television, the Institute presented 52 weekly science programs on WRCV–AM and a series on WHYY–TV featuring work in the Laboratories.

The twenty-one evening meetings of the Institute maintained a high level of interest and were attended by approximately 6100 persons. Eleven of the meetings were held jointly with other technical societies in the area, and the subjects ranged throughout the broad field of engineering and the physical sciences.

The Library, with its almost unsurpassed collection of technical books and literature, pamphlets and patents, continued to be of inestimable value to research scientists, both in the Delaware Valley and, by mail and telephone, in states as far away as Texas and California. Thus, although serving mainly in the Delaware Valley, this facility forms a real link between our first sphere of influence and the second, the Nation.

The Institute has, almost from the beginning, played a part in the scientific life of the Nation. In recent years, however, this role has assumed larger proportions, mainly because of the able and, at times, brilliant work of the Institute's Laboratories, and through the series of technical symposia and conferences sponsored by the Institute, all of which attained national recognition. Another facility which serves a wider area than the Delaware Valley is the recently established Computing Center.

The Computing Center drastically reduced its charges during the year as a result of which more companies are availing themselves of the services offered. While the workload in the Center was not heavy for the better part of the year, there was evidence of a turn in the trend before the year's end.

Two of the technical symposia in particular drew audiences from all over the country, namely, the Space Medicine Electronics Symposium, and, held in Washington, D. C., the Gas-Lubricated Bearings Symposium. A classified Navy briefing meeting attracted 450 representatives of U. S. industry, and a

conference on research and development opportunities in Government was of value to small business firms.

The Laboratories for Research and Development not only had a good year, volume-wise, but made many worthwhile contributions to the welfare of the Nation, both directly and indirectly. Contracts received during 1959 passed the four million dollar mark for the first time. New capital equipment was added, including the latest model electron microscope and the Varian instruments used in studies of nuclear and electron spin resonances.

Concentrating in four broad areas of research, namely, human welfare, basic investigation, industrial research, and research for Government, the Laboratories contributed many ideas, designs, devices, instruments and publications. Of those which are not classified and to which reference may thus be made, the following will best illustrate the breadth of scope: a study of odor and taste in public water supplies, the development of a pneumatic autopilot for business executive-type aircraft, advisory work on electroexplosive problems in the Polaris missile program, the servo-steering system in the Jupiter missile, improvement of the centrifuge used to train U. S. astronauts, and the design and construction of an advanced type of satellite.

In addition to these new projects, work continued on several long-range programs, such as air pollution, air traffic control, and nuclear engineering. With this varied program, the Laboratories' influence spreads throughout the Nation, and through work performed for the Government, beyond the national borders.

The Franklin Institute is known around the world for two of its activities which date back almost to its founding, namely, the Committee on Science and the Arts and the JOURNAL OF THE FRANKLIN INSTITUTE. *In recent years, the work of another organization, for which the Institute serves as Trustee, has made its name known abroad—The Biochemical Research Foundation, which, as part of its cancer research program, in 1959 successfully isolated a unique streptomycin-dependent strain which will be used as a screening organism in a search for agents which might reduce the incidence of cancer.*

The Committee on Science and the Arts, in its year-in, year-out search for men of achievement, recommended eighteen awards for presentation at the Medal Day ceremonies in October. The Franklin Institute's world-recognized Medals are bestowed regardless of the nationality of the recipient.

In addition, the Board of Managers in 1959 awarded the Vermilye Medal, for outstanding achievement in the field of Industrial Management, to George Romney, President of the American Motors Corporation. Mr. Romney received the award at the Stated Meeting in November, when the 135th Anniversary of The Franklin Institute was celebrated.

The JOURNAL OF THE FRANKLIN INSTITUTE *was distributed in 51 foreign countries, 48 of the states, and the District of Columbia. Among its research papers, several were related to satellites and to the problems connected with outer Space research. Thus the* JOURNAL *leads us on into the Institute's fourth and most distant sphere of influence, the Universe.*

The Bartol Research Foundation of The Franklin Institute has for many

years been a leader in the study of cosmic radiation in the upper atmosphere. In 1959, a new cosmic ray monitor was constructed for installation at McMurdo Naval Air Facility in Antarctica, to complement the one in operation at Thule, Greenland. Furthermore, the satellite "Explorer VII" carried into orbit a Bartol-designed detector of heavy nuclei. On an international level, the Bartol Foundation made significant contributions to two fields of physics dealing with nuclei and with solids. Closer to home, we wish particularly to record a healthy and mutually valuable cooperation between Bartol and Swarthmore College.

The factor which, in the final analysis, limits the scope and strength of The Franklin Institute's contributions to science is the amount of money available to carry out the planned programs.

The Board emphasizes that the Institute's Laboratories and the Bartol Research Foundation are self-supporting. Income from research contracts and, in the case of Bartol, from endowment funds, as well as grants, covers all expenses, including the purchase of new equipment. But, the Computing Center suffered a severe, but predicted, reduction in its workload for 1959 and thus shows a correspondingly large net operating loss for the year. It is expected that by the end of 1960, this activity will again have become self-supporting.

The preliminary financial figures for 1959 indicate that the Educational and so-called General activities of the Institute, which are scheduled for expansion to meet the growing need, will soon face a critical financial situation which can be averted through a concentrated and effective drive for funds. In 1959, a total of approximately $204,000 was contributed in the form of gifts, bequests and appropriations from Friends of Franklin, estates, foundations, corporations, the School District of Philadelphia, and the Commonwealth of Pennsylvania. This is an increase of about $19,000 over 1958. The net loss for Educational and General activities, although reduced substantially from the 1958 figure of about $100,000, was still over $20,000 for 1959, and will climb again as planned expansion gets under way.

The Board, in concluding its Report, feels strongly that the science education program must be expanded to meet the demands of the new decade, rather than curtailed to the level of present income. The Institute obviously cannot be expected to operate its educational programs continually at a loss, yet the need and demand for these programs are ever increasing. The only practicable solution is to appeal to the generosity of its members and friends, for funds to ensure not only the continuation, but the expansion, of the Institute's unique services to science education.

These, then, are the highlights of accomplishments of The Franklin Institute in 1959, in its outward-spiralling spheres of influence. The Board gratefully acknowledges the untiring efforts of the staff who performed the work and the loyal support of the members who are the Institute.

Respectfully submitted,
By order of the Board of Managers,

WYNN LAURENCE LEPAGE
President

January 20, 1960

Introduction

The year 1959 marked the 135th anniversary of the founding of The Franklin Institute and the 25th year of its fine edifice on the Parkway in Philadelphia. In presenting the Annual Report for 1959, the Board of Managers notes that important progress has been made in fulfilling the promise of the organization founded so modestly in 1824 for the mutual benefit of a few citizens of Philadelphia. In contradistinction, the sphere of influence of The Franklin Institute in 1959 extended from Pole to Pole, from West to East around the World, and far out into Space.

The founding fathers, whose principal aim was the promotion of the mechanic arts among themselves, would be amazed at the present scope of the Institute's science promotion activities, which in 135 years has grown to global proportions, covering a wide range, from young to old in age, from beginners to professionals in experience, and from teaching to research in method.

In this Report we present the highlights of 1959 in the form of a spiral, the focal point of which is The Franklin Institute in Philadelphia. The unbroken and ever-widening curves of the spiral reach out through the Delaware Valley, the Nation, the World, and the Universe beyond our planet.

Since our activities originate at the Institute, Delaware Valley naturally feels the impact of all of them. This Report, however, deals with each activity within the geographical area for which its contributions to science are the largest. For example, our Laboratories, though performing valuable research for local firms, are dealt with under "The Nation," because a greater volume of their work is done for the Government.

The Franklin Institute

Serves . . .

. . . THE DELAWARE VALLEY

Residents of Philadelphia and Delaware Valley are very nearly exclusive beneficiaries of five of the Institute's activities: the Science Museum, the Astronomy Department, Educational Radio and Television, the Meetings program, and the Library. Hence, in the first sphere of influence, this Report deals with these five only, even though many of the other activities are well known in this area. To take advantage of the opportunities offered through these activities, those wishing to learn must, in general, come to the Institute's headquarters where the facilities are located.

The Science Museum

The 410,433 people who visited the Science Museum during its 25th anniversary year found a wealth of exhibits about basic science, engineering, and industrial processes. Opened twenty-five years ago to instruct and interest young people and adults in the fundamentals of basic and applied science, the Museum has kept its exhibits up-to-date by making constant changes. Of the sixteen permanent exhibits added during the year, the most spectacular are the full-size "Matador" guided missile located outside on the southeast terrace, and the Smoke Tunnel to be seen in the Hall of Aviation. Three exhibits were loaned by the U. S. Patent Office for temporary display, and an exhibit of student research work sponsored by the Heart Association was on display for three months.

Over the years, the Museum has broadened its program of science education for young people to include career forums, Science Fairs, Summer and Winter Workshops, lecture-demonstrations, and the Dodge Lectures. All of these special features in 1959 showed marked gains in attendance over those in previous years. The two Career Forums—one sponsored jointly by the Institute and the Philadelphia Patent Law Association and the other by the Institute, the American Institute of Electrical Engineers and the Institute of Radio Engineers —attracted 494 high school students, all potential scientists and engineers of the future. The two Science Fairs exhibited the work of 983 students to no less than 27,000 visitors. The Summer Workshop, with an enrollment of 290 students, offered eight courses, one of which featured three field trips to study problems in astronautics. The Winter Workshop attracted 121 students. The 34th Annual Dodge Lecture was delivered on October 28 and 29 to a total audience of 630. A new lecture-demonstration called "The Science of Music"

For Young People in Delaware Valley

Some of the nearly 20,000 visitors who came to see the more than 400 entries in the Senior Science Fair, April 11–20, 1959.

Students in the Summer Workshop "Introduction to Science" course, watching a chemical demonstration.

Dr. Gilbert Haight performing a spectacular demonstration during the 1959 Dodge Lecture on "Fire."

Betsy Kraus, of Landenberg, Pa., the Gold Medal Winner in the Primary Division of the 1959 Junior Science Fair, shown with her winning exhibit, "The Candy Tree."

was first presented in the spring and repeated in November, when 2678 students attended the eight demonstrations in the second series. Another lecture-demonstration added in 1959 was entitled "Three Laws of Newton" and successfully given four times.

Two new educational programs were begun in 1959. A conducted tour of the Institute's Laboratories was arranged for high school students and proved so successful that it is to become a permanent part of the educational program. Secondly, the Museum inaugurated a program in cooperation with the Board of Education of the School District of Philadelphia which promises to be one of the Institute's most significant contributions to science education. Under this program, regular public school classes come to the Institute during school hours to study different phases of science under a certified teacher. In this way, the Museum's unique exhibits are used to best advantage in supplementing the science work done in the schools. The forty-one classes given during November and December covered four topics and were attended by 1388 students.

Approximately 290,000 school children visited the Museum in 1959. Attendance records for the past five years (see page 340) demonstrate the increasing interest in science evidenced by young people and adults alike. The yearly increase in attendance is mute tribute to the Institute's ability to satisfy that interest.

The Astronomy Department

In the Planetarium, six months in all were devoted to the subject of space travel to take full advantage of public interest in this topic. The demonstration, "Our Biggest Satellite—the Moon," enjoyed phenomenal success during September and October, for it opened just one day after Lunik II impacted the moon and was still showing when Lunik III was launched. The Department presented its annual lecture series on "Astronomy for Everyone" in the spring, and as evidence of the outward-spiralling influence of even this activity of the Institute, more than 1200 queries on astronomy and space travel, coming from all over the country, were answered. Members of the staff participated in the solar eclipse expedition in October, although bad weather precluded photographing the phenomenon.

Several exhibits were displayed through the efforts of the Astronomy Department, including a Vanguard exhibit and a model of the 600-foot radiotelescope being built in West Virginia. It is of interest to note that the Institute's Laboratories worked on the bearing designs for this great radiotelescope, bearings which support 2500 tons in weight, on an oil film 0.005 of an inch thick.

It is worthy of report that the Amateur Astronomers of The Franklin Institute resumed publication of "The Observer," a newsletter about astronomical events, and the Amateur Telescope Makers continued their pursuit of this fascinating hobby in their headquarters at the Institute. Eighteen new members joined during the year.

Educational Radio and Television

For the benefit of school students in the area, a weekly series of radio programs over WRCV–AM was presented by the Institute. These programs covered various science subjects and were used in many classrooms throughout the area as a supplement to the regular science work.

In the fall, six programs originating at WHYY–TV were televised to schools in the area. On these programs, Laboratories' personnel discussed different aspects of research, both basic and applied. These programs gave young people an opportunity to learn, first hand, the meaning of "research." WHYY–TV is Philadelphia's educational television station which was founded several years ago under The Franklin Institute's sponsorship.

Meetings

The Meetings program is an important phase of the scientific and educational activities of the Institute. Through this program, the latest technical information is made available to those professional people who seek to broaden their scientific knowledge in fields other than their own specialty, and to laymen who have a mature, intelligent interest in the broad fields of the physical sciences and engineering. Ten of the twenty-one evening meetings in 1959 were for Institute members; the remaining eleven were joint meetings with other technical societies in the area. A total of 6100 people attended these meetings during 1959. The table on page 339 reveals not only the variety of subjects presented, but also the excellence of the speakers, each of whom is an eminent authority.

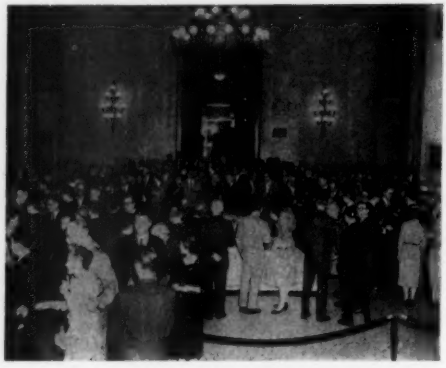
The Library

The accelerated pace of scientific research has posed many problems for the research Library. Foremost is the increased amount of scientific publication which has taken place since World War II and the inevitable demand by research workers for libraries to acquire, organize and make the material available. The Franklin Institute Library continues to provide this type of service.

Possessing a collection considered to be one of the finest in the United States for scholars and research workers in the fields of science and technology, the Library plays an important part in the industrial and research community of Delaware Valley. During 1959, 4281 volumes of journals, books, monographs, and patents were added, bringing the total to 167,135. Thirty-one new subscriptions strengthened the periodical holdings and increased to 1336 the number of journals currently received, including nineteen Russian journals and fourteen English translations of Russian serials. In addition, a number of rare patents were acquired through gift to improve this segment of the collection.

New equipment added during the year improved the services. These im-

For Adults in Delaware Valley



Franklin Hall Lobby filled with members attending the reception preceding the dinner on March 25, 1959, when the American Institute of Architects held a Joint Meeting with The Franklin Institute.

Wynn Laurence LePage, Institute President, congratulates Dr. W. F. G. Swann, at the Stated Meeting on December 16, 1959, when Dr. Swann delivered the Redding Lecture.

Pepper Hall, reading room of The Franklin Institute Library.

provements included an inter-com system, installation of higher wattage bulbs in the stacks, and purchase of a duplicating unit for rapid copying of material, especially for Laboratories' needs.

The 5021 people who used the Library in 1959 requested 13,585 titles and 5581 telephone inquiries were handled from as far away as California and Texas. In addition, 16,029 photostats were supplied, many to firms in other states. Groups of college students toured the Library and a number of Swiss educators, a member of the Russian Academy of Sciences and several distinguished scientists and librarians from various parts of the world visited the Library.

Thus, although serving mainly in the Delaware Valley, this facility forms a real link between the Institute's first sphere of influence and the second, the Nation.

. . . THE NATION

The Institute has, almost from the beginning, played an important part in the scientific life of the Nation. In recent years, however, this role has assumed larger proportions, mainly because of the able and, at times, brilliant work of the Institute's Laboratories, and through the series of technical symposia and conferences sponsored by the Institute, all of which attained national recognition. Another facility which serves a wider area than the Delaware Valley is the Computing Center.

Two symposia in particular drew audiences from all over the country. At the Space Medicine Electronics Symposium held in the spring, top authorities in this new field discussed the latest developments in Man's efforts to prepare a safe environment for his voyages into space. In the fall, the first international symposium on the important subject of Gas-Lubricated Bearings was managed by the Institute and held in Washington, D. C. In both instances, members of the Institute's staff contributed papers.

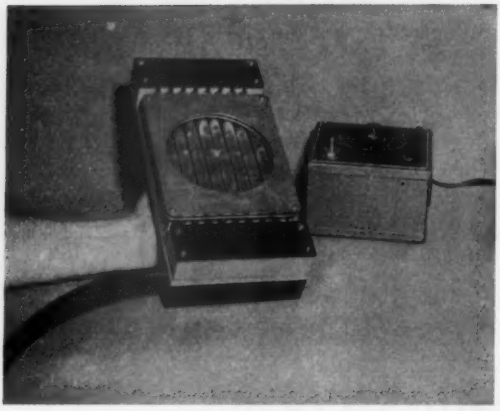
A classified Navy briefing was held at the Institute in October. This session on antisubmarine warfare and oceanography attracted 450 representatives of U.S. industry. In November, a conference on research and development opportunities in Government was of value to representatives of small business firms.

The Computing Center

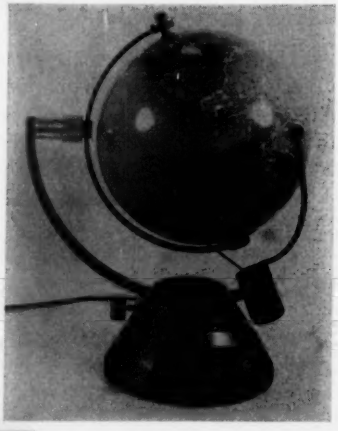
The Computing Center completed its third year of operation in 1959 and its first year as a separate activity. During the year, the Center drastically reduced its machine-use charges as a result of which more companies are availing themselves of the services offered.

Equipment modifications were made so that the UNIVAC, located in the Institute's main building, is now compatible with UNIVAC File computers.

Research Contributions



Radiant skin-burning apparatus, developed for medical researchers.

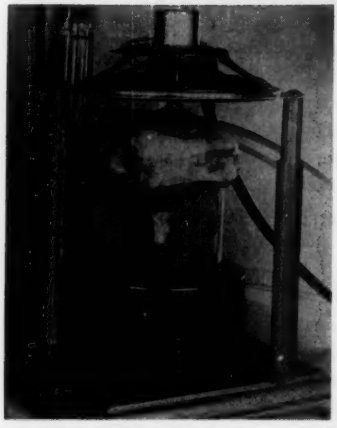


Orrery demonstrating the relative motions of a satellite and the Earth.

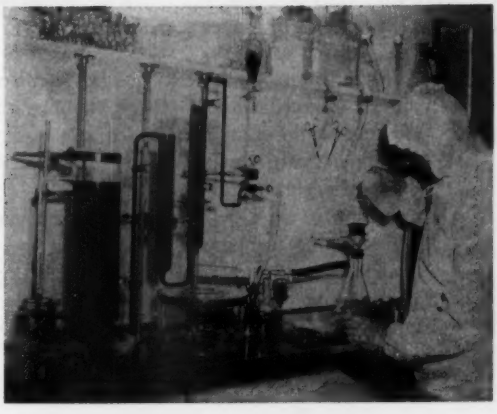
Flow loop being built for nuclear studies.

For the Nation

Pilot plant for coating plastic films to produce a hydrophilic surface for use in solar stills.



Experimental set-up for measuring resonance of the eye as related to intra-ocular pressure—a phase of the glaucoma research program.



Contamination studies of hydraulic servosystem that steers the Jupiter missile.

The Center's installation was designated as the official back-up facility for one such File computer activity. Other modifications speeded up and improved the Center's computing system.

In the fall, the Center offered an on-the-job programming course which was also open to representatives of industry. In this way outside firms are being taught to program their own work for eventual running on the Center's UNIVAC. The Institute's facilities were used in November for a week-long seminar on "Critical Path Scheduling," a new technique in electronic computer applications. Attendees, making use of the technique, ran problems on the Center's computer.

In addition to projects for the Laboratories, the Center's staff of analysts and programmers worked on thirty-four other projects during the year and completed nineteen of them by the end of the year.

The Laboratories for Research and Development

The Laboratories for Research and Development, throughout 1959, demonstrated their ability to develop basic services and establish new skills in keeping with the varied demands of industry and Government. Not only did they have a good year, in point of volume of work performed, but they made many worthwhile contributions to the welfare of the Nation, both directly and indirectly.

Contracts received during 1959 passed the four million dollar mark for the first time. New capital equipment was added, including the latest model electron microscope and the Varian instruments used in studies of nuclear and electron spin resonances.

The engineering and scientific programs in 1959 show achievement from which the Institute can derive much pride. Concentrating in four broad areas of research—human welfare, basic investigation, industrial research, and research for the Government—the Laboratories contributed many ideas, designs, devices, instruments and publications. Of those which are not classified and to which reference may thus be made, the following projects undertaken in 1959 will best illustrate the breadth of scope:

- A study of odor and taste in public water supplies
- A device for medical researchers, enabling them to produce radiant burns such as would occur in a nuclear blast
- Development work on an artery pulse pickup to detect early stages of cerebral occlusions
- Production of an ultra-pure iron bar one inch in diameter, by improved zone-refining techniques
- An apparatus to measure speed by detecting the light of an ionized gas at two points along the travel of the wave
- Preparation for the Atomic Energy Commission of a text book on thermal stresses in nuclear reactors

- Development of a pneumatic autopilot for use in business executive-type aircraft
- Advisory work on electro-explosive problems in the Polaris missile program
- Development work on coating plastic domes of solar stills, to enable them to collect condensate without impairing their ability to transmit sunlight
- Development of a Vertical Scale Indicator for tracking missiles during flight
- Improvement of the centrifuge used to train U. S. astronauts
- Design and construction of an advanced type of satellite
- Design of very large bearings for the radiotelescope being constructed in West Virginia
- Measurement of contamination of the servosystem that steers the Jupiter missile.

In addition to these new projects, work continued on several long-range programs, such as air pollution, air traffic control, design and analysis of reactor vessels and other structures for use in industrial nuclear power programs, engineering psychology studies, and structure of metals.

The program on gas-lubricated bearings is one example of these long-range studies. In this important field, the use of gas instead of liquid for lubrication of bearings greatly extends the useful range of temperature and speed within which bearings can function. The Laboratories direct a Government cooperative to perfect gas-lubricated bearing technology. Some accomplishments of this program have been the compilation and publication of the first full bibliography on the subject, management of an international symposium in October, and development work using nitrogen as a lubricant for a turbine compressor rotor to be used in a nuclear flow loop.

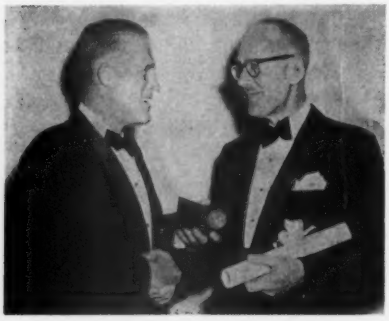
Through staff papers presented at international gatherings, through staff publications in technical journals which are read in many countries, and through work performed for the Government but destined to be used beyond the borders of the United States, the Laboratories carry our spiralling influence beyond the Nation, into the World.

. . . THE WORLD

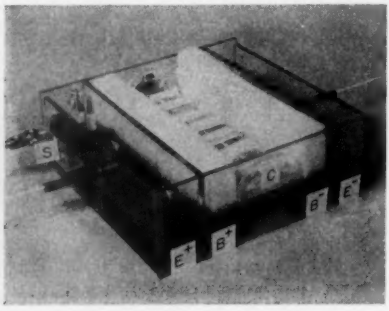
The Franklin Institute is known around the world for two of its activities which date back almost to its founding, namely, the work of the Committee on Science and the Arts and the JOURNAL OF THE FRANKLIN INSTITUTE. Furthermore, the work of The Biochemical Research Foundation, for which the Institute serves as Trustee has, in recent years, also made its name known abroad.

The World of Science and Industry

Group photograph of 1959 Medalists. At lower left, Mr. LePage, Institute President, examines Franklin Medal with Dr. H. A. Bethe, its recipient. First row (l. to r.): Dr. John Hays Hammond, Jr., Dr. Clarence M. Zener, Prof. Morris Cohen, Dr. George W. Morey, Dr. Irving Wolff, David M. Potter, Jr., and Jacob Rabinow. Second row (l. to r.): Dr. George G. Oberfell, Henry Charles Harrison, Robert B. Aitchison, Cyrus R. Osborn, and Dr. Charles H. Townes. Missing from the photograph are Dr. A. Hoyt Taylor, Dr. A. T. James, Dr. A. J. P. Martin, Dr. R. L. M. Synge, and Prof. Hardy Cross (deceased). *(For identification of medals received, see page 26.)*



George Romney (left), President of American Motors Corporation, receives the Vermilye Medal from Wynn Laurence LePage at the November Stated Meeting.



Electrophoresis apparatus used for isolating compounds of cancer-cell extracts in the study of the effects of treating cancer cells with 2-deoxyglucose.

Awards

The Committee on Science and the Arts, whose original purpose was to encourage inventors and to evaluate inventions, began in 1875 to grant awards for meritorious achievements by scientists and engineers. Over the years, the number of medals has grown from one to fourteen and the impartial and painstaking work of the Committee has become internationally famous. These awards, bestowed without regard to nationality, are recognized the world over as evidence of outstanding achievement.

At impressive Medal Day ceremonies in October, the work of this Committee of seventy-eight dedicated members culminated for the year in the award by the Institute of medals to seventeen individuals and one company, for outstanding work in many fields (see page 340).

In addition, the Board of Managers in 1959 awarded the Vermilye Medal for outstanding achievement in the field of Industrial Management to George Romney, President of the American Motors Corporation. Mr. Romney received the award at the Stated Meeting in November, when an illustrious audience dined in Franklin Hall to celebrate the 135th Anniversary of The Franklin Institute and to meet the distinguished medalist.

The Biochemical Research Foundation

In its concentrated and many-facetted attack on cancer, the Biochemical Research Foundation in 1959 advanced the campaign against this dread disease. One of its achievements was the successful isolation of a unique streptomycin-dependent strain of *E. coli,* sought for three years. This strain will be used as a screening organism in a search for antimutagens, agents that might reduce the incidence of cancer. Another achievement was the discovery of a new "isoantigen" in the rat which controls the transplantation of normal as well as malignant rat tissues. This research may lead to an explanation of the spontaneous regression of tumors.

Long-term studies of the effects of nutrition on cancer growth were continued, as were studies of the metabolic processes of cancer cells, and of the effects of treating cancer cells with 2-deoxyglucose, an inhibitor of tumor respiration.

In the long, arduous fight to conquer cancer, the Biochemical Research Foundation has contributed much of value to research workers all over the world, for cancer recognizes no national boundaries.

Journal of The Franklin Institute

THE JOURNAL OF THE FRANKLIN INSTITUTE, which completed 134 years of continuous publication in December, 1959, is one of the Institute's most widely known contributions to science, for it is distributed in 51 foreign countries, 48 of the states, and the District of Columbia. The JOURNAL serves as

a publication outlet for theoretical and applied research workers of many nationalities and in many fields of science and engineering. Illustrative of the non-specialized character of the JOURNAL are the following fields, covered in some of the 44 technical papers published during the year: astronautics, electronics, dynamics, mechanics, mathematics, geophysics, civil engineering, psychology, ballistics, solid state and theoretical physics. Among these, several dealt with problems connected with outer Space research and satellites. Thus the JOURNAL leads us on into the Institute's fourth and most distant sphere of influence, the Universe.

. . . THE UNIVERSE

To the founders of the Institute, the "Universe" consisted of the Earth, which they knew something about, and the "Heavens," of which they knew little. Today, Man's immensely greater knowledge of the Universe is the result of scientific advances in optics, mathematics, physics, aerodynamics and astronomy. The Institute is proud of the fact that its Laboratories for Research and Development and its Bartol Research Foundation have made many contributions to this greater body of knowledge, which will ultimately lead to Man's conquest of outer space.

Bartol Research Foundation

The Bartol Research Foundation of The Franklin Institute has long been a leader in the study of cosmic radiation in the upper atmosphere. In 1959, Bartol's cosmic ray group participated in the International Geophysical Cooperation, the sequel to the International Geophysical Year. This group constructed a new cosmic ray monitor for installation at McMurdo Naval Air Facility in Antarctica, to complement the one in operation at Thule, Greenland, thus extending our influence from "Pole to Pole." Measurements of cosmic ray intensities were also made with airborne and shipboard monitors in several round-the-world surveys. In collaboration with the Martin Company, Bartol scientists designed the detector of heavy nuclei which, borne aloft aboard the satellite Explorer VII, is expected to provide data throughout the one-year lifetime of the transmitter.

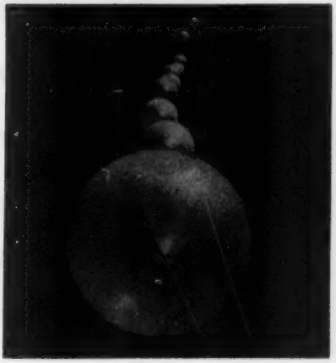
This work parallels contemporary work in our Franklin Laboratories, where research on the design, construction and operation of equipment used in missiles and satellites—and even the satellites themselves—is currently furthering Man's knowledge of the universe.

In addition to these contributions to our knowledge of the Universe, the Bartol staff has collaborated with physicists of foreign nations, and a number of countries including India, Japan, Holland, Israel and Greece are represented among the post-doctoral fellows at Bartol.

Toward Greater Understanding of the Universe

Pile structure, heart of the cosmic ray neutron monitor designed for installation in the Antarctic.

Satellite-borne equipment presently operating aboard Explorer VII. Assembled unit without potting material is at upper left. The three decks of transistorized circuitry in the foreground were developed by RIAS Division of the Martin Company and the heavy nuclei detector by the Bartol Research Foundation in a collaborative undertaking.



Balloons launched by the Bartol staff to carry instruments into the upper atmosphere.

On the national level, research programs for the U. S. Government were carried out in the fields of nuclear physics, physical electronics and solid state physics.

On the local level, mutual cooperation has flourished with Swarthmore College, on whose campus Bartol's headquarters are located. An active program of scientific colloquia and public lectures has been initiated, and undergraduate students at Swarthmore have been encouraged to participate in Bartol's research program.

The widespread program at Bartol is typical of other activities of the Institute—centered in Delaware Valley, their influence spirals out, carrying the name of The Franklin Institute throughout the Nation, the World and the Universe.

Organization

The Franklin Institute is a non-profit organization, composed of members who elect a Board of Managers to determine general policies for the Institute and guide its operations. The Board elects the Officers, who are responsible for general supervision and management. Day-to-day activities are carried out by the staff, under the direct supervision of heads of departments, who may or may not be officers. Because of the nature of the Institute, it must depend upon outside sources for funds to augment the proceeds from its limited endowment and with which to meet its educational responsibilities.

Board of Managers and Officers

The Board of Managers records with deep regret the death of one of its valued members and a Past President of the Institute, Charles S. Redding, who passed away January 2, 1960. J. Frank Cox, Joseph A. Fisher and Joseph Gray Jackson were appointed to fill unexpired terms on the Board.

In January, the Director of Laboratories was made an officer of the Institute. The resignation of John S. Burlew as Executive Vice President and a member of the Board of Managers was regretfully accepted in November. These were the only changes in Officers during the year.

Members

In keeping with the original purposes of the Institute, its members today are engaged in many professions and businesses. Their common interests are a desire to know more about science and technology, and a wish to support the worthwhile educational activities of the Institute. Although the majority of members live in Delaware Valley, the sun literally never sets on Institute members, for some live in such far-away places as Brazil, Denmark, South Africa, South India and Russia.

On December 31, 1959, there were 7354 members compared to a total of 7037 at the end of 1958. These members showed great interest in those activities of The Franklin Institute which were presented especially for the membership. These activities included the monthly Sunday Movies of a popular scientific nature, the special parties to "Meet the Staff," and the "Science Is Fun" meetings.

T. Edward Ross signs the Institute's guest book at the dinner honoring the 1959 Gold Members. Looking on, left to right, are Theobald Clark, Morton Gibbons-Neff (Chairman of the Membership Committee), Fred H. Colvin (who became a member in 1888), and James McGowan, Jr. Messrs. Clark, McGowan and Ross are 1959 Gold Members.

At the dinner preceding the May 20th Stated Meeting, the four new Gold Members for 1959 were honored: Theobald F. Clark, James McGowan, Jr., T. Edward Ross and, in absentia, Walton Clark, Sr. At the December 16th Stated Meeting, the forty-five Silver Members for 1959 were honored. The Board is proud to report that the Institute had 17 Gold Members and 238 Silver Members at the end of the year.

Staff

On December 31, 1959 there were 532 employees, as compared to 545 on December 31, 1958. Of these, seventeen have been with the Institute for twenty-five years or more.

The deaths of three employees are recorded with deep repret: Edward J. Dougherty, Assistant Receiving Clerk; Edward Boyle, Maintenance Porter; and James J. Fisher, Museum Guard.

During the year six employees were retired: Bernard Hare, Maintenance Porter; Wilbur G. Jackson, Maintenance Engineer; Frank Hoffman, Museum Guard; Joseph Frega, Maintenance Electrician; Kathryn Read, Museum Cashier; and Dr. W. F. G. Swann, Director of the Bartol Research Foundation.

Under the Institute's Education Aid Plan, 51 employees received refunds totalling $3668.00 for college tuition fees. Many of these people received degrees for graduate and undergraduate work successfully completed under this plan.

Nineteen service pins were awarded to employees who, during 1959, became eligible for these awards, as follows: one (Walter A. R. Pertuch, Librarian) for 55 years; one (William F. Jackson, Jr., Secretary) for 35 years; four for 25 years; four for 15 years; and nine for 10 years.

The Kite and Key Club, the employee social club organized in 1958, held many successful functions during the year, including dance parties and group trips to baseball and football games. Small groups participate in such activities as bowling, bridge, chess, fencing and photography. Pool and ping-pong are enjoyed by many during their lunch periods.

Walter A. R. Pertuch, Librarian, who completed 55 years of service at The Franklin Institute in 1959.

Hostess Committee

The Hostess Committee, a group of prominent Philadelphia women, again graciously served as hostesses during the receptions preceding Institute dinners. Particular thanks are due the Committee for the beautiful floral decorations at Medal Day, and for their continued efforts to beautify the Institute's grounds.

Public Relations

With the growing need for increased education in science and engineering, it becomes more important to project the image of the broad scientific and educational work of the Institute. This is being accomplished through the public relations group, whose two principal functions are (1) advising and counseling Institute personnel in the planning of programs, and (2) exercising every effort to promote these programs among industry, the academic world, and those people who are interested in knowing more about the scientific and engineering developments that play such an important part in their everyday lives.

In order to keep the public informed of its affairs, the Institute maintains a Public Relations Office. Here, press releases are prepared for newspapers and magazines, arrangements are made for press, radio and TV coverage of important events, and requests for information about the Institute are handled.

In addition, *The Institute News* is published eleven times a year. Although intended primarily for members, this four-page newspaper is also distributed to schools, libraries and other organizations interested in science education.

Finances

The factor which, in the final analysis, limits the scope and strength of The Franklin Institute's contributions to science is the amount of money available to carry out the planned programs.

The Board emphasizes that the Institute's Laboratories and the Bartol Research Foundation are self-supporting. Income from research contracts and, in the case of Bartol, from endowment funds as well as grants, covers all expenses, including the purchase of new equipment. During 1959, the Computing Center suffered a severe, but predicted, reduction in its workload and thus shows a correspondingly large net operating loss for the year. The over-all results of operation since its inception three years ago, however, are in the black and it is expected that by the end of 1960, this activity will again have become self-supporting.

The financial statements of the Institute are audited each year by Messrs. Lybrand, Ross Bros. and Montgomery. When their report of audit is completed, together with explanatory notes, it will be available at the Treasurer's office for inspection by members.

The preliminary financial figures for 1959 (see page 341) indicate that the Educational and so-called General activities of the Institute, which are scheduled for expansion to meet the growing need, will soon face a critical financial situation which can be averted through a concentrated and effective drive for funds. In 1959, a total of approximately $204,000 was contributed (see page 341) in the form of gifts, bequests and appropriations from Friends of Franklin, estates, foundations, corporations, the School District of Philadelphia, and the Commonwealth of Pennsylvania. This is an increase of about $19,000 over 1958.

With reference to the Educational and General activities, it is difficult to realize a substantial increase in income from these sources, although a rise in admission rates for the Museum produced an additional $35,000. This, together with slight increases and non-recurring items in other income, and carefully controlled expenses, reduced the net loss for Educational and General activities substantially from the 1958 figure of about $100,000 to approximately $20,000 for 1959. This net loss, however, will climb again as planned expansion gets under way.

Conclusion

The Board, in concluding its Report, feels strongly that the science education program must be expanded to meet the demands of the new decade, rather than curtailed to the level of present income. The Institute obviously cannot be expected to operate its educational programs continually at a loss, yet the need and demand for these programs are ever increasing. The only practicable solution is to appeal to the generosity of its members and friends, for funds to ensure not only the continuation, but the expansion, of the Institute's unique services to science education.

These, then, are the highlights of accomplishments of The Franklin Institute in 1959, in its outward-spiralling spheres of influence. The Board gratefully acknowledges the untiring efforts of the staff who performed the work and the loyal support of the members who *are* the Institute.

Respectfully submitted,
By order of the Board of Managers,

WYNN LAURENCE LEPAGE
President

TABLES OF FACTS AND FIGURES

INSTITUTE MEETINGS IN 1959

	Date	Title	Speaker
A	January 21	*The Response of Man to His Total Environment*	Rene J. Dubos
J	January 28	*Billion Volt Accelerations*	G. Kenneth Green
J	February 4	*The American News Picture—How We Get the Greatest Impact*	Harold Blumenfeld
S	February 18	*Now It's Got to Work*[1]	H. Thomas Hallowell
J	February 25	*The Impact of IGY on Science and Engineering*	Hugh Odishaw
J	March 4	*Ballistic Missile Early Warning System*	Charles B. Joliffe
S	March 18	*Research from Fundamentals to Fulfillment*[2]	Augustus B. Kinzel
J	March 25	*The Air Force Academy*	Walter A. Netsch, Jr.
J	April 1	*The Youngest and the Oldest Stars of Our Galaxy*	Bengt Strömgren
S	April 15	*The Greatest Invention of Them All*[3]	Henry Belin duPont
J	April 22	*Tomorrow's Agriculture—The Research Perspective*	Byron T. Shaw
J	May 6	*Tomorrow's Weather*	Francis K. Davis, Jr.
S	May 20	*The Space Market*[4]	Edward B. Doll
F	September 30	*Why Sports Cars?*	Stephen F. Wilder
J	October 7	*75 Years—A Prologue to the Future*	Brig. Gen. David Sarnoff V. Adm. John Hayward
S*	October 21	*Fundamental Particles*	Hans A. Bethe
SP	October 28	*Franklin's River Today—The Story of the Deep Delaware*	Lt. Gen. E. C. Itschner
J	November 4	*The Aluminum Overcast—Traffic Jam in the Sky*	Capt. A. C. Packard
S	November 18	*Is America Losing Sight of Its Direction?*[5]	George Romney
J	December 2	*The Fuel Cell—The Most Direct Road to Electricity*	H. A. Liebhafsky
S	December 16	*The Science of Yesterday, Today and Tomorrow*[6]	W. F. G. Swann

A—Annual Meeting.
J—Joint Meeting.
S—Stated Meeting.
F—Science Is Fun.
SP—Special Meeting before "Ports of Philadelphia Day."
* Medal Day.

1—Philip C. Staples Endowed Lecture.
2—William B. Coleman Endowed Lecture.
3—Edward G. Budd Endowed Lecture.
4—Charles Day Endowed Lecture.
5—Celebration of 135th Anniversary and presentation of Vermilye Medal.
6—Charles S. Redding Endowed Lecture.

FIVE-YEAR RECORD OF MUSEUM ATTENDANCE

Year	Children *	Adults	Total
1955	219,765	92,248	312,013
1956	238,207	98,063	336,270
1957	263,305	114,304	377,609
1958	275,258	130,601	405,859
1959	289,452	120,981	410,433

* Up to and including those of senior high age.

AWARDS PRESENTED AT MEDAL DAY, OCTOBER 21, 1959

Medal	Recipient	Subject
Franklin	Hans Albrecht Bethe	Thermonuclear studies
Cresson	John Hays Hammond, Jr.	Remote radio control
Cresson	Henry C. Harrison	Matched impedance in electromechanical systems
Cresson	Irving Wolff	Centimeter wave field and microwave radar
Potts	George W. Morey	Rare earth glasses
Ballantine	Albert Hoyt Taylor	Radio communication developments
Ballantine	Charles H. Townes	Maser
Brown	Hardy Cross *	Moment-Distribution Method
Clark	George G. Oberfell	Liquid Petroleum Gas
Clamer	Morris Cohen	Heat treatment of steels
Henderson	Electro-Motive Division of General Motors Corporation	Diesel-electric locomotives
Wetherill	A. T. James	Gas chromatography
Wetherill	A. J. P. Martin	
Wetherill	R. L. M. Synge	
Wetherill	Robert B. Aitchison	Jet-piercing equipment for mining
Wetherill	Clarence M. Zener	Solid state physics
Longstreth	David M. Potter, Jr.	Rotary flow meter
Longstreth	Jacob Rabinow	Magnetic fluid clutch

* Awarded posthumously.

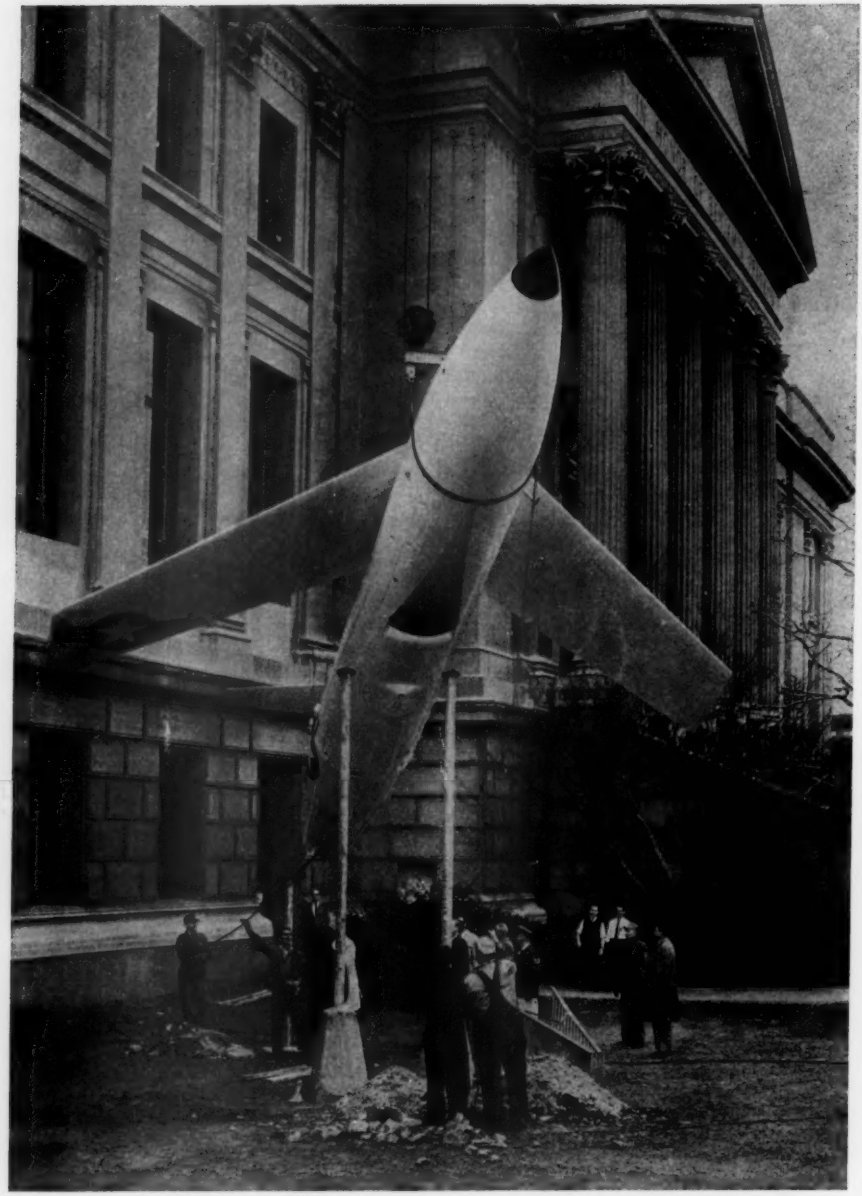
FUNDS RAISED DURING 1959

GENERAL OPERATIONS	
Friends of Franklin	$ 62,415.83
Appropriations	
Commonwealth of Pennsylvania	61,759.00
Board of Public Education, School District of Philadelphia	9,000.00
ENDOWMENTS	
Unrestricted	
Estate of Sarah Maude Kaemmerling	33,541.64
Estate of Charles S. Redding	9,000.00
William Messinger Memorial Fund	2,582.25
GIFTS FOR SPECIFIC PURPOSES	
Dolfinger-McMahon Foundation	7,500.00
W. Maxwell Scott, Jr.	4,943.33
D. Robert Yarnall	100.00
18 individuals and companies (for Smoke Tunnel)	4,847.88
Charles E. Hires Co.	1,100.00
Philadelphia Foundation—Truitt Memorial Fund	600.00
Hall Planetary Co.	50.00
Contributions for Basic Chemistry Exhibit	3,928.30
Industrial cooperation for Paper Exhibit	3,240.00
TOTAL	$204,608.23

COMPARATIVE TABLE OF INCOME AND EXPENSES FOR EDUCATIONAL AND GENERAL ACTIVITIES

	*1959 **	*1958*	*1957*
INCOME			
Membership Dues	$ 73,400	$ 68,077	$ 67,986
Science Museum and Planetarium	248,900	208,679	194,345
Journal	32,000	32,264	33,400
Endowment Income	127,000	121,742	118,340
Friends of Franklin	60,900	56,714	53,123
State Appropriation	62,300	58,850	65,016
Board of Education	9,000	9,000	9,000
Other Income	76,900	36,417	52,253
Total income	$690,400	$591,744	$593,463
EXPENSES			
Science Museum and Planetarium	$442,400	$430,181	$408,544
Library	109,300	105,486	102,858
Journal	41,400	46,318	42,353
Learned Society Activities **	85,100	87,073	—
250th Anniversary	—	—	13,168
General Expenses	32,700	23,379	57,731
Total expenses	$710,900	$692,437	$624,654
Net Loss	$ 20,500	$100,693	$ 31,191

* The figures for 1959 are projected and are subject to year-end adjustments and audit.
** The expenses for these activities (Membership, Meetings, Science and the Arts and Fund Raising) were included, prior to 1958, in General Expenses and prorated departmentally.



The Matador missile, gift of the U. S. Air Force, being lifted into its permanent location on the southeast terrace of the Institute's grounds.

THE FRANKLIN INSTITUTE

MINUTES OF THE STATED MEETING

March 16, 1960

The Stated Meeting of The Franklin Institute was held at 8:15 P.M. in the Lecture Hall. Mr. Wynn Laurence LePage, President, called the meeting to order with approximately 210 members and guests present.

The President stated that the minutes of the Annual Meeting of January 20, 1960 were published in the February issue of the JOURNAL. There being no additions or corrections the minutes were approved as published. He stated that the minutes of the Stated Meeting of February 17, 1960 will be published in the next issue of the JOURNAL and will be presented at a later Stated Meeting of The Institute for approval.

The President then announced with much pleasure that the Boyden Premium would be awarded this evening. The Boyden Premium, established in 1859, is awarded not oftener than once in five years to any resident of North America who has recently made a notable experimental determination of the speed, in free space, of radiation in any region of the entire spectrum. The Committee on Science and the Arts has recommended, and the Board of Managers approved, the award of the Boyden Premium in 1960. He then called on Mr. Lewis P. Tabor, a member of the Committee on Science and the Arts, who presented Captain Carl I. Aslakson, U.S.C.G. (Ret.), the recipient of the Boyden Premium in 1960. The citation reads "in consideration of his contribution to the measurement of the speed of radiation in space through the use of Shoran techniques and thereby as the first American to aid in establishing a new and significantly more nearly accurate value of 16 km per second higher than the long accepted value." The President presented to Captain Aslakson the Boyden Premium in the amount of $500. and the Certificate and Report. Captain Aslakson expressed deep appreciation for this award.

President LePage noted that this was the occasion of the Seventh Edward G. Budd Lecture, this Lecture having been established in 1948 by the family of the late Edward G. Budd as a tribute to his long-time membership in The Institute and on its Board of Managers. The Lecture is endowed with the purpose of keeping members of The Institute informed of new ideas in science and engineering. The President then introduced the speaker, Mr. Donald G. Fink, Director of Research of the Philco Corporation, who spoke on "Infrared—The Spectral Frontier." Mr. Fink presented a paper reviewing the principles underlying the generation, transmission and detection of infrared energy and described typical applications in science, industry and military pursuits. Mr. Fink illustrated his talk with slides and presented a demonstration of a new infrared scanning device developed in the Philco Research Laboratories.

Following a discussion period, President LePage expressed in behalf of the members of The Institute his warm thanks to Mr. Fink for his talk which was well confirmed by the enthusiastic response of the audience.

The meeting was adjourned at 9:40 P.M.

WILLIAM F. JACKSON, JR.
Secretary

COMMITTEE ON SCIENCE AND THE ARTS

(Abstract of Proceedings of Stated Meeting held Wednesday, March 9, 1960.)

HALL OF THE COMMITTEE,
PHILADELPHIA, MARCH 9, 1960.

MR. J. PHILIP EVANS in the Chair.

The following report was presented for final action:

No. 3336: Aerocar.

This report recommended the award of an Edward Longstreth Medal to Moulton B. Taylor, of Longview, Washington, "For his conception, design, development and construction of a successful flying automobile which is completely roadable and has been fully certified for both ground and flight operation."

D. S. FAHRNEY,
Secretary to Committee

MEMBERSHIP

The following elections were approved on March 10, 1960 by the Membership Committee by authority given to it by the Board of Managers.

ANNUAL

George J. Alles
Karl V. Amatneek
Edward Blackman
George C. Buzby
John D. M. Cameron
Frank S. Crossley
William E. Denk
Charles J. Devlin, Jr.
Walter M. Fishl
John H. Geyer
I. W. Gower
James L. Grim
Cornelius R. Guinee
Ernest A. Harding
Frederick C. Heller
Frank J. Heymann
Geoffrey S. Holister
Marshall G. Holloway
Kyonghi Hong
Henry Horner
T. J. Huston
Homer W. Johnson
James M. Kelly
Sidney Kleiman
Semond Levitt
Leonard E. Liss
John J. Magyarik
Irving J. Maitin
Richard H. Marvin
H. Donald Moll
C. Calvin Naylor
John K. Nevin
Margaret A. Oliver
Thomas O'Malley
Martin A. Pomerantz
Joseph H. Proctor
James T. Purcell
Jacob Ravitz
Joseph E. Reger
Robert C. Rietheimer
Howard M. Roseman
Harry H. Rosen
Waldemar Saeger
Richard F. Scharmann
George S. Schreiber
Alfred C. Schroeder
Nathaniel E. Schwartz
George W. Schwenzer
Rabah A. Shahbender
Ralph J. Shapcott
Bernarr J. Sharf
William B. Shepard
C. Bradford Sheppard
Nathan Shuman
Marvin Silver
George J. Silvestri, Jr.
Chester M. Sinnett
Walter Sinton
J. Paul Skelly
Stanley Skutnick
Evard Snell
Philip R. Snyder
Boris H. Traven
August J. Vella
Edward S. Weyl

EDUCATOR

Raymond S. Berkowitz
Andrew J. Demes
Chester Dudziak
E. F. Jenkins
John D. McCrumm
A. Klair McGlynn
Donald S. Pearson
Philip J. Sentner
Irwin Siegelman
William Weiss

NON-RESIDENT

Paul F. Pagerey

NECROLOGY

Charles S. Cheston '36
Robert M. Green, Jr. '36
John B. Hendrickson '49
S. Earle Moore '52
William A. Sloan '39
Edith A. Turner '36
John C. Taylor '36

BOOK REVIEW

THE THEORY OF THIN SHELLS, by V. V. Novozhilov, translated by P. G. Lowe, and edited by J. R. M. Radok. 376 pages, diagrams, $6\frac{3}{4} \times 9\frac{1}{4}$ in. Groningen, P. Noordhoff Ltd., 1959. Price, $9.50.

The basic analytical tool of this book, which is devoted to the linear theory of thin shells, is the method of complex transformations initiated by Meissner and extended by the Russian school of elasticians. By expressing the equilibrium forces and the displacements of a shell element in complex form, and making certain simplifications, it is possible to reduce the number of differential equations to be solved from six to three. The conciseness of this representation is claimed to simplify transformations into more tractable forms, and appears to be a great advantage when the solution involves Fourier series, which may be used in their complex form.

In the first chapter, the author develops an outline of this method on the basis of the differential geometry of surfaces, and the equilibrium and continuity conditions of a shell element, concluding with a demarcation of the range of validity of the solution. The second chapter begins with a lucid exposition of the range of applicability of the membrane theory, and applies it to the analysis of domes and pressure vessels, including shells of constant strength and shells subjected to wind loading. The results are extended to domes with discontinuous supports, and shapes obtainable from axisymmetric shells by affine transformations. The concluding paragraphs introduce the application of membrane theory to cylindrical shells.

The third chapter deals with the analysis of cylindrical shells, in which the moments and normal shearing forces are taken into account. The complex form of the equation is applied to the analysis of cylindrical shells reinforced by circumferential stiffeners; of cylindrical plates, and of shells composed of several cylindrical plates with individually constant radii of curvature. The results are simplified for the case of long cylindrical plates. Extensive tables of coefficients for stresses and displacements should make this section valuable to the designer. The chapter concludes with a brief exposition of the "semi-membrane theory" of long cylindrical shells, which neglects the moment tending to deform the generator, and the torque tending to twist the middle surface. The resulting simplification is justified by the assumption that the variation of stresses and deflections is likely to be more gradual along the length of the cylindrical shell than in the circumferential direction.

The final chapter treats shells of revolution. The complex equations for the displacements are applied to cases of axisymmetric loading. An approximate method of asymptotic integration is used to derive influence coefficients for the deformation of the edge of the shell. Further subjects include pressure vessels with dished heads of various forms, an extensive analysis of toroidal shells, and a discussion of curved end panels which contains one of the few worked-out examples in the book, as well as extensive tables. A discussion of wind loading, including some general design formulae, and of spherical shells subjected to arbitrary loading, concludes the chapter.

Many of the results presented are given in Timoshenko's "Plates and Shells" in a form somewhat more condensed, and perhaps more readily answering to the needs of the engineer. But while the theoretical bias of Novozhilov's book is indicated by its title, the tabular and graphical material contained in the second half should make it useful to the designer. In any case, in presenting the analysis of shells from a unified point of view, and in clearly outlining the limitations of the theories employed, it should appeal to engineers to whom the theoretical elasticity serves as a stimulus and guide for more rational design. The text is well written, and the typography and figures are particularly clear and attractive.

K. L. CAPPEL
Franklin Institute Laboratories

BOOK NOTES

PHYSICS OF THE EARTH'S INTERIOR, by Beno Gutenberg. Vol. 1 of the International Geophysics Series. 240 pages, diagrams, 6 × 9 in. New York, Academic Press, Inc., 1959. Price, $8.50.

Embracing a wide range of properties and processes, this volume on the Earth's interior aims to give as complete a picture of the present status of this controversial science as possible. Conclusions and hypotheses concerning our planet's interior are in a state of flux and some basic conclusions which ten years ago were considered to be practically certain, in recent years have been found incorrect. Many other fundamental problems, such as the origin of the Earth, its thermal history, temperature, composition and the state of the core, or the so-called 20°-discontinuity in the mantle are just as controversial as they were a few decades ago. Consequently, in many sections of this book the unsolved problems that are pointed out are more numerous than the reports of results which were apparently accomplished. The nine chapters include data on the Earth's structure, its crust, mantle, core, temperature, density, pressure, gravity, elastic constants and processes in the Earth. The author, with his colleague, Ernst Wiechert, in 1914, through seismograph measurements, determined the sharp division between the core and the mantle begins about halfway between the surface and the center of the Earth. It is still referred to as the Wiechert-Gutenberg discontinuity.

HANDBOOK OF SUPERSONIC AERODYNAMICS. SECTION 16, MECHANICS OF RAREFIED GASES, by Samuel A. Schaaf and Lawrence Talbot. 89 pages, diagrams, 8 × 10¾ in. Washington, Government Printing Office, 1959. Price, $1.25 (paper).

Section 16 of Volume 5 of the Handbook of Supersonic Aerodynamics, published for the Bureau of Ordnance, gives quantitative and reliable information for the case of a gas in which rarefaction effects are just becoming evident (slip flow) and for the case of a completely rarefied gas (free molecular flow). For these two cases, lift, drag, and heat transfer data for typical bodies are presented in the form of calculations and experimental results. Various flow regimes of rarefied gas dynamics are discussed and defined.

CONTRIBUTIONS TO THE THEORY OF GAMES. VOLUME IV, edited by A. W. Tucker and R. D. Luce. 453 pages, diagrams, 7 × 10 in. Princeton, Princeton University Press, 1959. Price, $6.00 (paper).

Dedicated to the memory of John von Neumann, this final volume in the "Contributions to the Theory of Games" series is devoted to recent work in *n*-person games. Nineteen papers include treatises on symmetric solutions for zero-sum *n*-person games, on the four person game, the five person game, the Edgeworth Market games, on non-zero-sum games, on extreme games with three values, and one by von Neumann entitled "On the Theory of Games of Strategy."

PHYSICS OF THE ATOM, by M. Russell Wehr and James A. Richards, Jr. 420 pages, diagrams, 6 × 9 in. Reading, Mass., Addison-Wesley Publishing Co., 1960. Price $8.50.

This text is designed to extend the introductory college physics course into the realm of atomic physics, and to give the student a proficiency in this field comparable to that which he has acquired in mechanics, heat, sound, electricity and optics. The material is organized into logical series of topics leading to the practical conversion of mass energy into kenetic energy. The logical progression of topics is developed as simply and directly as possible: the problem is stated, the solution is presented, and the significance of the result is interpreted. Containing many illuminating analogies, this book will aid the student to bridge the gap between classical physics and the present frontiers of physical investigation.

INTRODUCTION TO RUBBER TECHNOLOGY, edited by Maurice Morton. 547 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $10.00.

This volume is the direct result of the growing educational activities of the various active Rubber Groups in the United States, and also represents one of the many traditional activities of the American Chemical Society's Division of Rubber Chemistry. A compilation of selected lectures that introduces new members of the rubber industry to the basic aspects of rubber technology is the substance of the book's broad coverage of a rapidly expanding field.

BEACHES AND COASTS, by Cuchlaine A. M. King. 403 pages, illustrations, 6 × 9 in. New York, St. Martin's Press, Inc., 1960. Price, $14.50.

Although dealing primarily with the coasts of Great Britain, this book includes examples taken from many countries. Until the publication of Dr. King's book, there had been no comprehensive publication about coastal processes since 1919. Twelve chapters deal with methods of research, waves, surveying techniques, wind effects, coastal accretion and erosion, and other phases of the subject. The book should be very useful to civil engineers who are engaged in research or planning on coastal problems.

NUCLEAR FUEL ELEMENTS, edited by Henry H. Hausner and James F. Schumar. 409 pages, illustrations, 6 × 9 in. New York, Reinhold Publishing Corp., 1959. Price, $12.50.

This is the collection of papers presented at the First International Symposium on Nuclear Fuel Elements, held in January of 1959, at Columbia University. The twenty papers cover a great variety of fuel elements (uranium, zirconium, thorium, plutonium, ceramic) and fabrication problems for high temperature elements, for APPR fuel plates, for the ISNSE element, and for low power research reactors. A valuable "extra" is the 24-page bibliography comprising Appendix II.

TURBULENCE, by J. O. Hinze. 586 pages, illustrations, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1959. Price, $15.00.

This text (one of the McGraw-Hill series on Mechanical Engineering) is intended to give the student enough of a theoretical background on turbulent fluid flow to enable him to appreciate the specialized literature on turbulence and to carry out engineering investigations such as mixing and heat and mass transfer. The seven chapters cover general concepts, measurement of turbulent flows, isotropic turbulence, nonisotropic turbulence, transport processes, nonisotropic free turbulence and nonisotropic "wall" turbulence. To each chapter is appended a complete nomenclature. The book should prove useful not only as a text, but as a tool for chemical and mechanical engineers.

THERMODYNAMIC PRINCIPLES FOR CHEMICAL ENGINEERS, by Roger Gilmont. 339 pages, diagrams, 6 × 9 in. New York, Prentice-Hall, Inc., 1959. Price: $11.00 (trade); $8.25 (classroom).

The author has written a basic text on thermodynamics especially for the chemical engineer. He characterizes chemical engineering thermodynamics as applied physical chemical thermodynamics, with emphasis on the flow process and the irreversible process. In order to avoid as much as possible the use of higher mathematics, the author employs graphical mathematics (graphical methods of calculation), although any student who hopes to understand this text must have a good background in mathematics through differential equations.

EDTA TITRATIONS, by H. Flaschka. 138 pages, 5½ × 8½ in. New York, Pergamon Press, 1959. Price, $6.50.

Prof. Flaschka, one of today's outstanding analytical chemists, has prepared this small but valuable collection of experiments using ethylene-diamine tetra-acetic acid. His experience as a teacher is reflected in the quality of the material included in this book, which will no doubt end up in every chemistry teacher's library, where it will be referred to again and again. Each experiment is com-

plete in itself and each has been selected to demonstrate a particular principle.

PLASTICS ENGINEERING HANDBOOK of the Society of the Plastics Industry, Inc. Third edition, 565 pages, illustrations, 7 × 10 in. New York, Reinhold Publishing Corp., 1960. Price, $15.00.

A completely revised, expanded and up-to-date third edition of this handbook on plastics engineering is the most useful source of information ever made available to one industry. There is much new material on nomenclature, cellular plastics, decorating, welding and plastics as adhesives. The book describes every step in the manufacturing operation, and includes the experience and know-how of over 200 technicians and authorities. Fully illustrated with many photographs, tables and charts which provide data of immeasurable value to the plastics plant.

RADIOISOTOPE TECHNIQUES, by Ralph T. Overman and Herbert M. Clark. 476 pages, diagrams, 6 × 9 in. New York, McGraw-Hill Book Co., Inc., 1960. Price, $10.00.

This laboratory textbook deals with the principles and practice of handling and measurement techniques that will enable the scientific investigator to use radioactive materials in his research program. A compilation of material on preparation of radioactive sources, laboratory characterization, standardization, and separation methods, as well as the practical aspects of radioiostope applications is included in this text. Its major purpose is to present the theory and principles underlying the techniques, along with the discussion of laboratory procedures. A set of typical problems is included as a teaching aid.

THE CONCEPTS AND THEORIES OF MODERN PHYSICS, by J. B. Stallo, edited by P. W. Bridgman. 325 pages, 5¼ × 8¼ in. Cambridge, Mass., Harvard University Press, 1960. Price, $4.75.

When this book on the fundamental concepts and general theories of late nineteenth-century physics was first published in 1881, it was far ahead of its time. It was the first, and for long the only extensive American book in the philosophy of science. In its French, German and British editions, it was one of very few texts in that field to appear in Europe. The American public of the '80s and '90s was not ready for it, but abroad its significance was recognized. Possibly the author's major contribution is his anticipation of the special role of the law of the conservation of energy. The book can be regarded as an important landmark in American intellectual history. This republication, with an extensive introduction by P. W. Bridgman, is a welcome addition to history of science libraries.

CEMENTED CARBIDES, by Paul Schwarzkopf and Richard Kieffer. 349 pages, diagrams, 6 × 9¼ in. New York, The Macmillan Company, 1960. Price, $15.00.

Cemented carbides make modern high-speed machining possible and find many other places where resistance against wear, corrosion and heat are important. The authors, recognizing a growing technical need for a comprehensive and up-to-date treatment of cemented carbides, have drawn upon sources in the United States and in Western and Eastern Europe in preparing this work. The opening chapters furnish background information and describe the methods of producing carbides and other metallic refractories. The middle chapters describe the mechanical and chemical properties of all the cemented carbides in commercial production and those being produced in experimental quantities. A chapter on the new oxide and boride cutting materials is included. Final chapters concentrate on the many applications of the cemented carbides to the machine tool, mining, defense and other industries.

CLASSICAL MATHEMATICS—A CONCISE HISTORY OF THE CLASSICAL ERA IN MATHEMATICS, by Joseph Ehrenfried Hofmann. 159 pages, 5 × 8 in. New York, Philosophical Library, Inc., 1959. Price, $4.75.

The broadly panoramic story of mathematics covering the seventeenth and eighteenth centuries—the periods known as the high and late Baroque and the Age of Enlightenment, is herein presented in this book. The author relates the development of mathematics to the growth of intellectual principles.

Some of the topics include the system of Descartes, the discovery of the power series, the invention of calculus and others which lead us directly to the threshold of the modern era.

THEORY OF UNIMOLECULAR REACTIONS, by Noel B. Slater. 230 pages, 6 × 9¼ in. Ithaca, Cornell University Press, 1959. Price, $4.75.

In this new text, the author treats the molecule as a completely analyzed vibrating system, of which the time to dissociation is determined by the vibrational behavior, so that both the absolute rate constants and the relative effects of concentration become calculable. This gives a purely mechanistic picture, free of what Prof. Slater calls "quasi-thermodynamics," and involves an analysis of the behavior of sums of vibrations. The ten chapters include ones covering first-order reactions both on the classical harmonic model and in transition-state theory, the theory of general and second-order rate constants, quantum harmonic oscillator models, and a chapter on a new approach to rate theory. The treatment encompasses parts of both the older Eyring and HKRR viewpoints of chemical kinetics. The book is based on a series of lectures given at Cornell University when the author was a guest lecturer there.

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Underwater Detection System.—The impact point of a ballistic missile or re-entry vehicle hitting the surface of the ocean can be located by underwater detection systems developed for the U. S. Navy by Bell Telephone Laboratories and installed by the Western Electric Company at both the Atlantic and Pacific Missile Ranges. The impact creates noise signals that are used to locate the spot where the missile hit the water.

Now in operation in conjunction with the Atlantic Missile Range, these underwater detection systems are aiding recovery teams in finding and retrieving missile nose cones after their space flights.

Several missile-impact-locating systems (MILS) stations have been established, including one on the British island of Ascension. Similar stations have been set up or are being constructed on islands in the Pacific for the Pacific Missile Range.

Two types of acoustic systems are used in locating the missile impact point. One is a surface-impact system and the other is a Sofar (Sound Fixing and Ranging) bomb method.

The surface-impact system is used for detecting and locating the sound of a missile actually striking the ocean's surface. This system uses six hydrophones, or underwater sound receivers, installed on the ocean floor and connected by a special cable to the shore station. Five of the hydrophones are located in the shape of a pentagon with the sixth receiver in the center. Information from at least three of the six hydrophones is needed to obtain an "acoustic fix." This provides an accurate "fix" over a restricted target area.

In the Sofar method a bomb is ejected from the missile and exploded underwater in the vicinity of the impact point. The sound of the exploding bomb is detected and measured to determine the area of impact. With Sofar systems, detection is possible at distances of several thousand miles.

For the Sofar detection method, a series of hydrophones, generally located in pairs, are spaced about a large area and connected to shore stations by submarine cables. Electronic equipment at the shore stations records the signals from the hydrophones when the sound waves generated by the explosion of the Sofar bomb are received. Trained operators at the shore installations are able to use the time differences of the sound arrivals at different hydrophones to obtain an "acoustic fix" that establishes the spot where the sound originated.

An operator must select the pairs of hydrophones to use for his calculations. The operator obtains the time of the signal arrival at each hydrophone in the various pairs of hydrophones he has selected. By subtraction this gives him a time difference between any two hydrophones. Since the location of the hydrophones and the velocity of sound in water at each location are known factors, the operator is able to convert time difference into distance difference by multiplying the time difference by the average velocity of sound for the combination of two hydrophones. Then he determines the acoustic fix by locating the

intersection of two or more distance-difference lines on previously prepared charts.

By combining data from various stations in the Atlantic Missile Range, it is possible to obtain acoustic fixes on impacts throughout the range.

The detection system also serves as a navigation aid to recovery vessels. Prior to a missile launching the ship drops Sofar bombs and uses the time of the signal arrival at the hydrophones to determine its exact position in relation to the expected impact point of the missile.

To arrive at an acoustic fix it is necessary to know the speed with which sound travels through water at the location of each hydrophone. This velocity of sound at any given point in the ocean is a function of temperature, pressure and salinity.

To check velocity and other factors in the Atlantic Ocean, a three-month calibration operation was conducted last year by personnel from Bell Telephone Laboratories, the Western Electric Company and the U. S. Navy. Some 1500 explosive charges were dropped in the ocean range areas to simulate the conditions of a missile-locating operation. By knowing the explosion time and position of these charges it was possible to check the accuracy of the underwater detection systems and provide data for the operators' use in determining acoustic fixes.

Although an operator at the missile-impact-locating station plots the acoustic fix on his charts, a more exact calculation is obtained by using a computer. Arrival time data from the Atlantic shore stations are sent to Patrick Air Force Base, Florida, and relayed to Winston-Salem, North Carolina, where the information is fed into a computer at Western Electric's plant. Precise determination of the impact point is furnished by this operation.

The Underwater Systems Development Department of Bell Telephone Laboratories at Whippany, New Jersey, developed the detection systems. Equipment for the systems is furnished and installed by Western Electric's plant in Winston-Salem.

Aluminum Salt Water Distillation Unit.—An all-aluminum sea water distillation unit, the first known application of aluminum for this purpose, is being developed by the U. S. Army Engineer Research and Development Laboratories, Fort Belvoir, Va.

Equipment currently supplied to troops for desalting sea water is fabricated of cupro-nickel. When development of the all aluminum unit is complete, the new equipment will be lighter, have greater capacity, better fuel economy, and will eliminate the requirement for large amounts of nickel which may not be available in the event of mobilization.

An experimental all-aluminum unit is currently undergoing engineer tests near Daytona Beach, Fla. Previously, it underwent limited testing at Wrightsville Beach, N. C., after which it was completely disassembled and examined for evidence of corrosion. Very little corrosion was noted and, after reassembly, the unit was then sent to the test site at Daytona Beach. After a total of 2000 hours of testing, the unit will again be carefully inspected to determine the extent of corrosion.

Use of aluminum instead of cupro-nickel required redesign of several features of the distillation units. The heat exchanger design now prevents accumulation of sludge in low flow areas. All copper was eliminated from the accessories, such as pumps, piping, and strainers because it was deter-

mined that even trace amounts of copper in sea water promote galvanic corrosion of the aluminum.

Sea water is pumped through heat exchangers where it is heated by the flows of distilled water and concentrated brine leaving the equipment. The sea water is boiled inside the tubes of a combination evaporator and condenser and the resulting vapor is drawn into a steam compressor where the pressure of the vapor is increased a few pounds per square inch with a corresponding rise in temperature. The compressed vapor is returned to the evaporator-condenser where it is condensed and transfers its heat to additional sea water. This vapor compression distillation process, which conserves the latent heat of vaporization, requires less fuel than any other distillation process developed at this time.

Final design of the equipment is expected to result in a trailor-mounted unit, suitable for delivery by helicopter, and capable of producing 125 GPH of potable water from sea water.

Plug Nozzle Engine.—General Electric has released details of a radically new rocket engine design concept promising substantial time and cost savings in the development of multi-million pound thrust propulsion systems. Called the "plug nozzle engine," the concept was originated by General Electric several years ago and has been the subject of intensive Company-funded investigation for some time, according to Louis Michelson, Manager of G. E.'s Rocket Engine Section.

The primary advantage offered by the design is its application flexibility. Once basic engine development is completed, Michelson explained, the design can be readily and inexpensively scaled to virtually unlimited multi-million pound thrust powerplants. The engine can be adapted to varying rocket requirements with a "substantial reduction in long and costly development required to build new propulsion systems," he said.

Thus far in the country's missile and space program, propulsion system designers have been faced with combustion phonemena which have largely defied logical scaling, he said. As the diameter of the fuel injector and thrust chamber of a conventional engine are increased to achieve higher thrust levels, severe oscillatory modes not present in the smaller unit are encountered. This and other scaling problems have necessitated a "cut and try" method with all meaningful testing conducted with combustors of a final thrust size. The results of this approach are long development cycles, sizable expenditures in both hardware and testing and large investments in costly high-thrust test facilities, Michelson pointed out.

Multi-million pound thrusts can be achieved through the use of a number of cellular combustion chambers which can be designed and tested on a small scale and then assembled to achieve desired thrust levels. The design makes it possible to keep combustor size small enough to suppress adverse combustion phenomena.

Basically an assembled configuration, the engine differs from clustered propulsion systems now under development in that its combustion cells would be designed specifically to be assembled into a single integrated power package.

In the plug nozzle engine, these combustor segments would be assembled to form a segmented annular combustor about the base of a conic-shaped spike or plug, resulting in a short, compact propulsion package. In contrast to conventional combus-

tion chamber designs, a large portion of the gas expansion occurs externally. These gases would be confined only by the plug surface, the outer jet surface remaining a free boundary.

Supersonic gases would expand as they flow along the conical plug surface until their pressure matched existing ambient conditions and would therefore be automatically adjusted to altitude, increasing the rocket's efficiency. This higher efficiency would be most pronounced at low altitudes, Michelson explained, making the engine particularly suitable for the initial stage of a multistage vehicle.

The multiple-cell configuration will permit in-flight trajectory correction by the control of combustion pressure in the various cells. Thrust can therefore be varied aerodynamically without moving the entire engine, allowing rigid mounting between propulsion system and vehicle frame. The resulting elimination of the usual gimbaling mechanisms and structures for stress redistribution makes possible additional significant savings in weight, space and costs. Michelson explained that the idea was conceived by Dr. Kurt Berman with others in his group at the Malta Test Station. Dr. Berman is Manager of Engineering for the Rocket Engine Section.

Non-Corrosive Anti-Freeze.—A new anti-freeze that will not corrode aluminum or other metal engine parts has been developed by Olin Mathieson Chemical Corporation. The new anti-freeze is non-evaporating and suitable for any type engine that uses a liquid coolant.

F. W. Fitchen, national brands manager of the corporation's Automotive Products Department, said that development work on the new formula anti-freeze was completed more than a year ago in Olin Mathieson's laboratories in Niagara Falls, and the product has been undergoing rigid tests ever since.

He said it was developed in anticipation of wider use of aluminum in automobile engines. Many 1960 cars have the metal in various engine parts, including cylinder blocks, cylinder heads, radiator cores, water jackets, water outlet housings and water pump parts.

Dr. Chester M. White, who heads Olin Mathieson's anti-freeze testing laboratories, said one of the problems in developing the new formula involved the development of a better corrosion inhibitor system. This in turn involved a correctly balanced formula. The correct balance of the new formula, Dr. White explained, results in an improved ethylene glycol antifreeze harmless to non-metallic parts and non-corrosive to all metals.

Halogen Leak Detectors Find New Use.—Halogen leak detectors—handy troubleshooters for the air conditioning and refrigeration serviceman—have found an entirely new application. They are helping keep laboratory animals germ-free. At Notre Dame University, where Lobund Institute researchers are studying the effect of micro-organisms and medications on animals, transparent vinyl chambers called "isolators" are used to establish a completely sterile environment.

Because the vinyl is thin and susceptible to breaks, a method was needed to assure each isolator "microbe-proof" before starting an experiment. As a result, three General Electric H-6 leak detectors are now in daily use.

To check an isolator, a research assistant fills it with Freon gas, and moves the detector probe carefully

over the entire surface. Microscopic holes in the plastic skin, identified by a flashing light in the probe, are marked immediately for patching.

Lobund researchers said that prior to purchasing the General Electric instruments, they also explored other methods of leak detection—soap films and halide torches. They said the H-6 leak detector was selected as having the best balance of economy and convenience, while actually offering more than enough sensitivity for their needs.

The effectiveness of the institute's germ-free research depends entirely upon its leak detection program, they said, since any contamination will cancel out efforts to maintain a sterile environment.

Lobund's use of Halogen leak detectors, General Electric engineers said, exemplifies a continually broadening range of applications for the low-cost device originally designed to find leaks in air conditioning and refrigeration equipment.

Tiny New Electron Tube.—Space vehicles, television sets, radios, TV cameras, and computors will soon use a unique, new thimble-size electron tube, according to Douglas Y. Smith, Vice President and General Manager, RCA Electron Tube Division.

The revolutionary RCA development in electron tube design, called the Nuvistor, is now rapidly progressing from advanced development to production status, Mr. Smith reported. Engineers, he added, are now building experimental TV receivers, TV cameras, and computing devices employing the tiny new tube.

"The Nuvistor," Mr. Smith explained. "is an extremely rugged, metal-shell electron tube about three-fourths of an inch in length which promises an extremely high level of performance and a degree of reliability never before anticipated from tubes produced in large quantities. The 'new look' tube offers miniaturization capabilities that can significantly increase payload capacities of military vehicles and commercial aircraft. For home entertainment instruments, the Nuvistor will make possible high-performance, lightweight, compact AM and FM radios, phonographs, hi-fi and TV sets.

"In laboratory torture tests, these remarkable tubes have continued to operate efficiently at both frigid and sizzling temperatures (from 320 degrees below zero to 660 degrees Fahrenheit). They can also withstand severe shock and vibration thereby making them extremely advantageous for ground or airborne military vehicles."

Mr. Smith pointed out that the Nuvistors have extremely low power requirements. The new design concept, he said, opens the door to even smaller tubes consuming as little as one twentieth the power required by conventional tubes. Engineers anticipate that tubes of the same design can have useful lifetimes of tens or even hundreds of thousands of hours.

"More than 250 manufacturers of electronic equipment are currently evaluating prototypes of the Nuvistors for use in TV sets, communications receivers and a wide range of applications for entertainment, defense and industry. Initial commercial production of Nuvistors is scheduled for this year."

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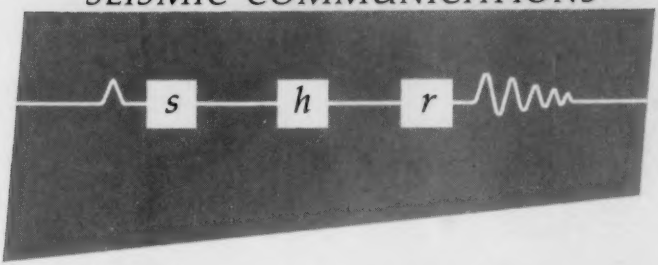

An evaluation of the use of rock strata as a means of information transfer is being conducted by Texas Instruments research. Theoretical studies, field experiments, model studies (illustration), aim at development of emergency, "hard" communications by seismic means.

Seismic communication is just one area where TI earth sciences are being applied to programs of military interest. Missilery . . . surveillance . . . submarine warfare . . . have developed needs covering the entire geophysical spectrum. Texas Instruments — through programs in seismology, gravimetry, oceanography, magnetics — supports these needs and extends the operational capabilities of weapons and anti-weapons systems.

For further information, request Bulletin DM-101.

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